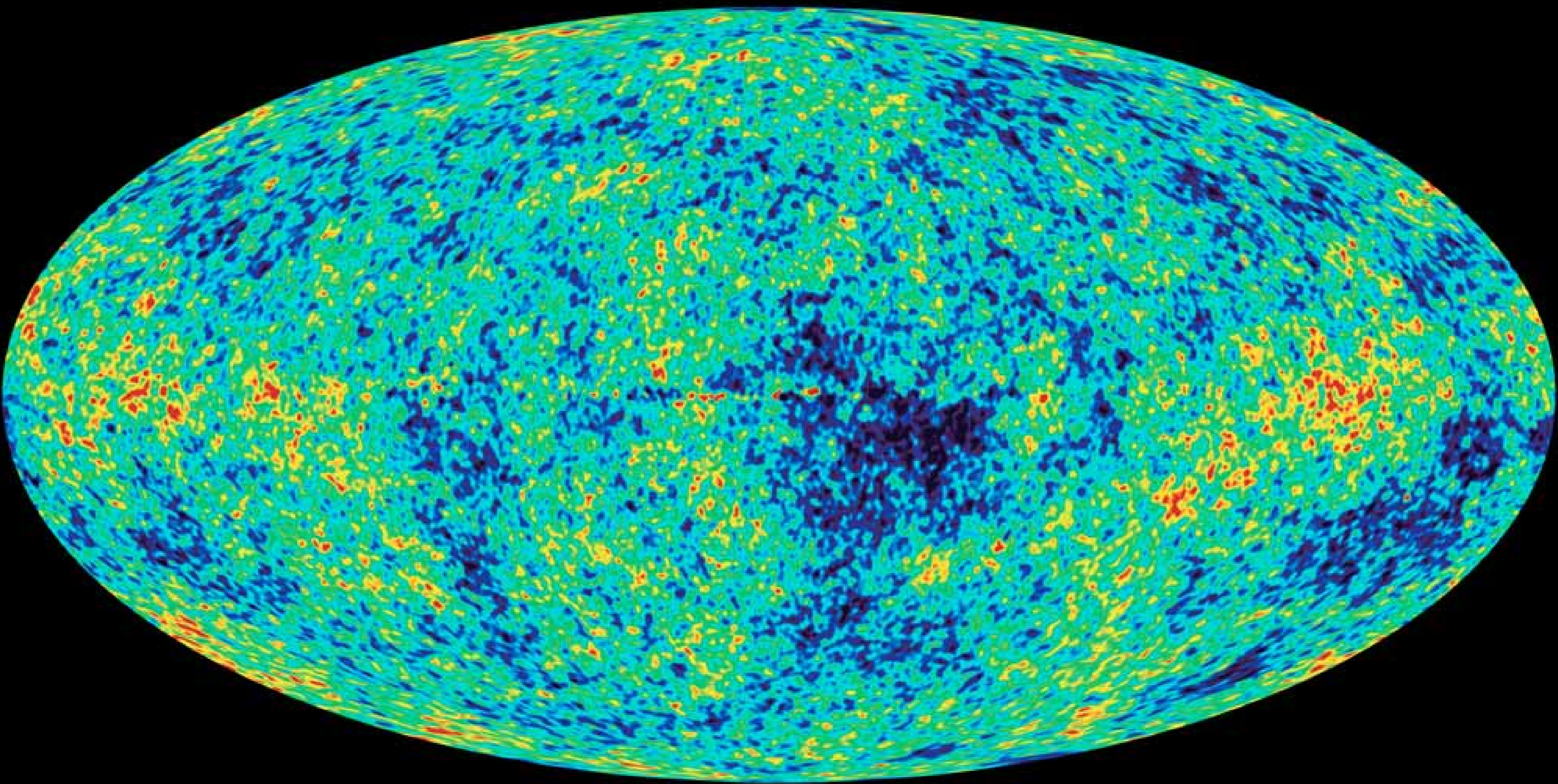
상대론적 우주구조의 비선형 진화

Nonlinear evolution of relativistic cosmological structures

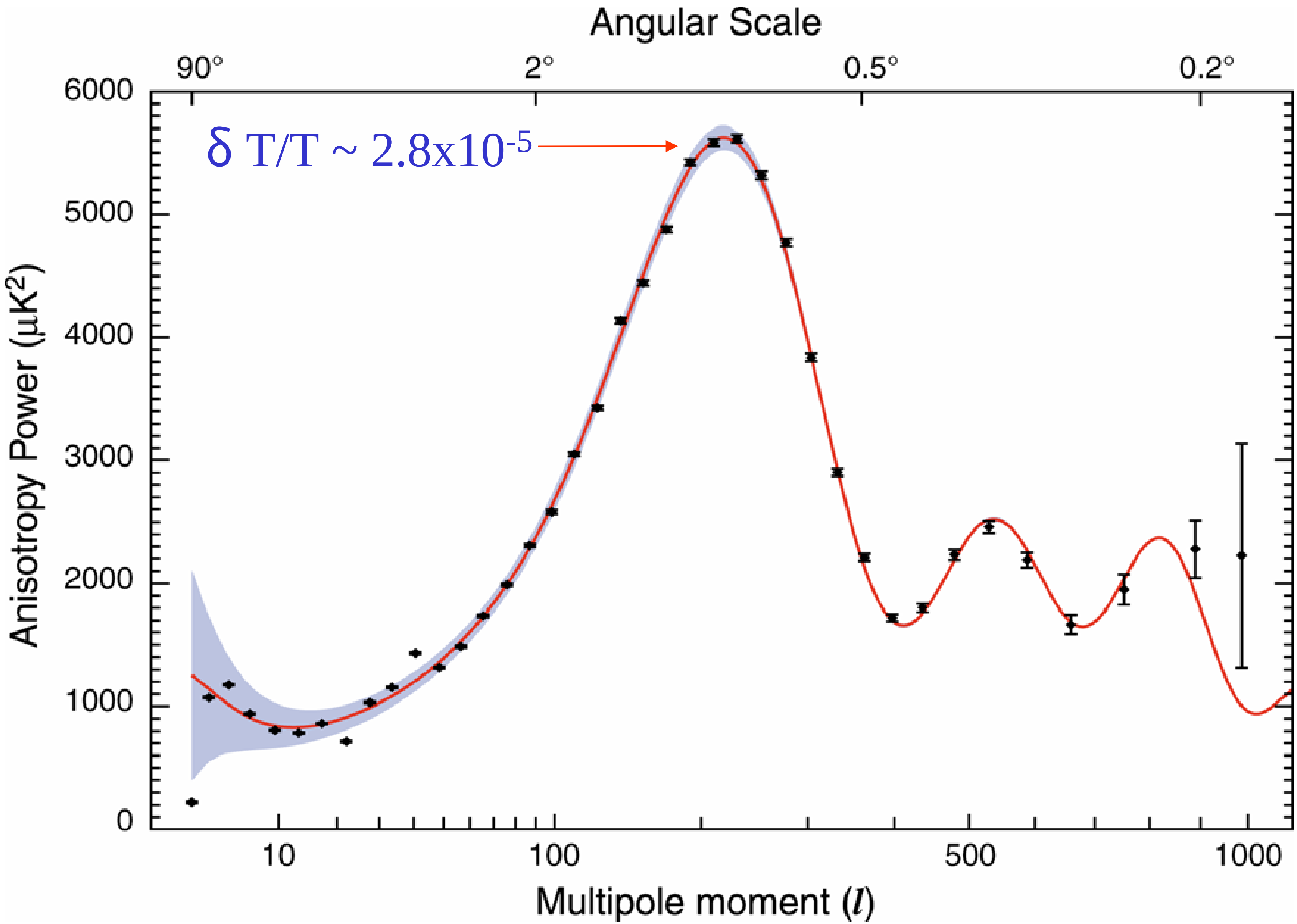
J. Hwang and H. Noh
KPS meeting
2007.04.20

CMB: linear structure $\delta T/T \sim 10^{-5}$



http://map.gsfc.nasa.gov/m_or.html

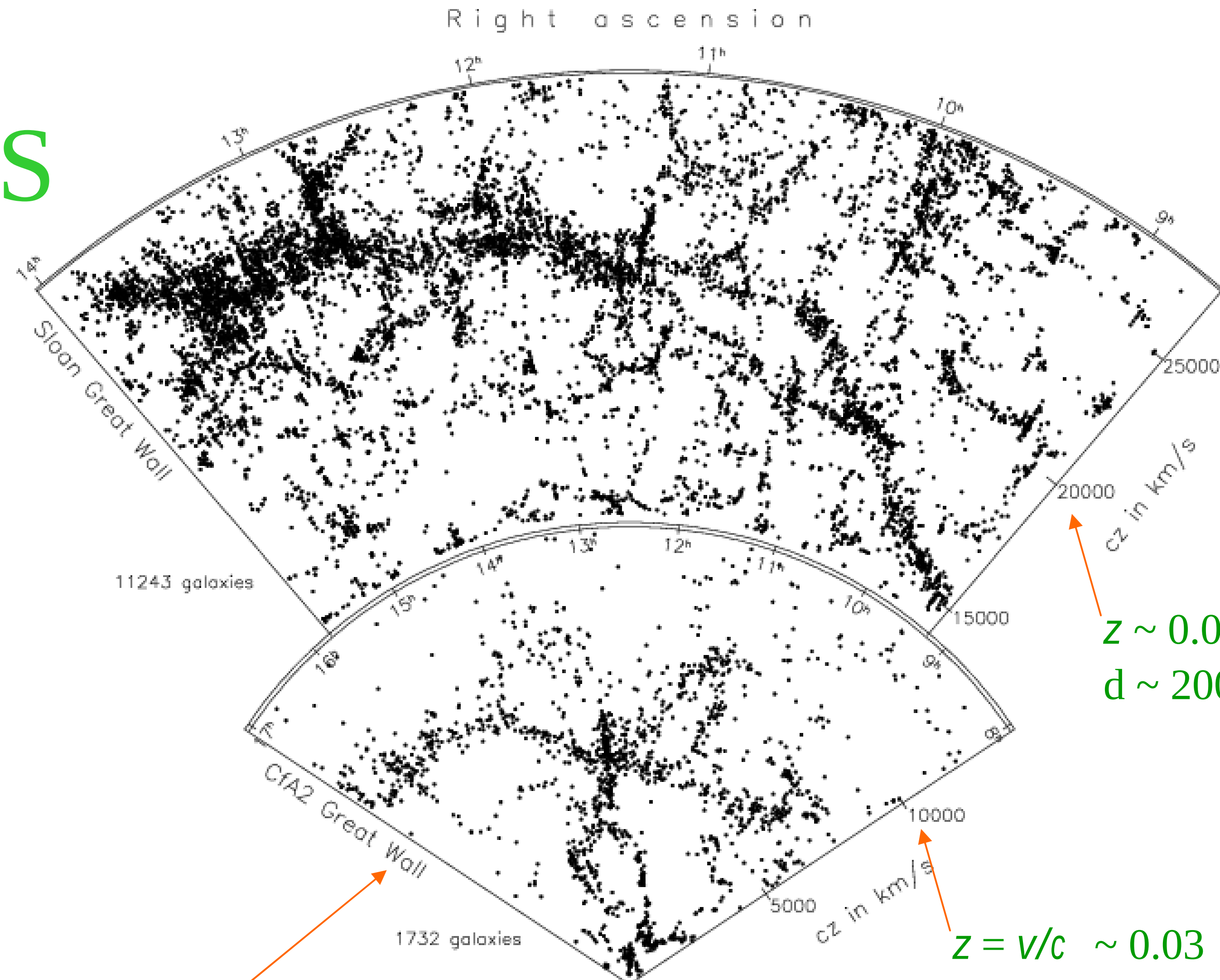
WMAP Temperature anisotropy power spectrum



http://map.gsfc.nasa.gov/m_mm.html

LSS: non-linear structure

SDSS



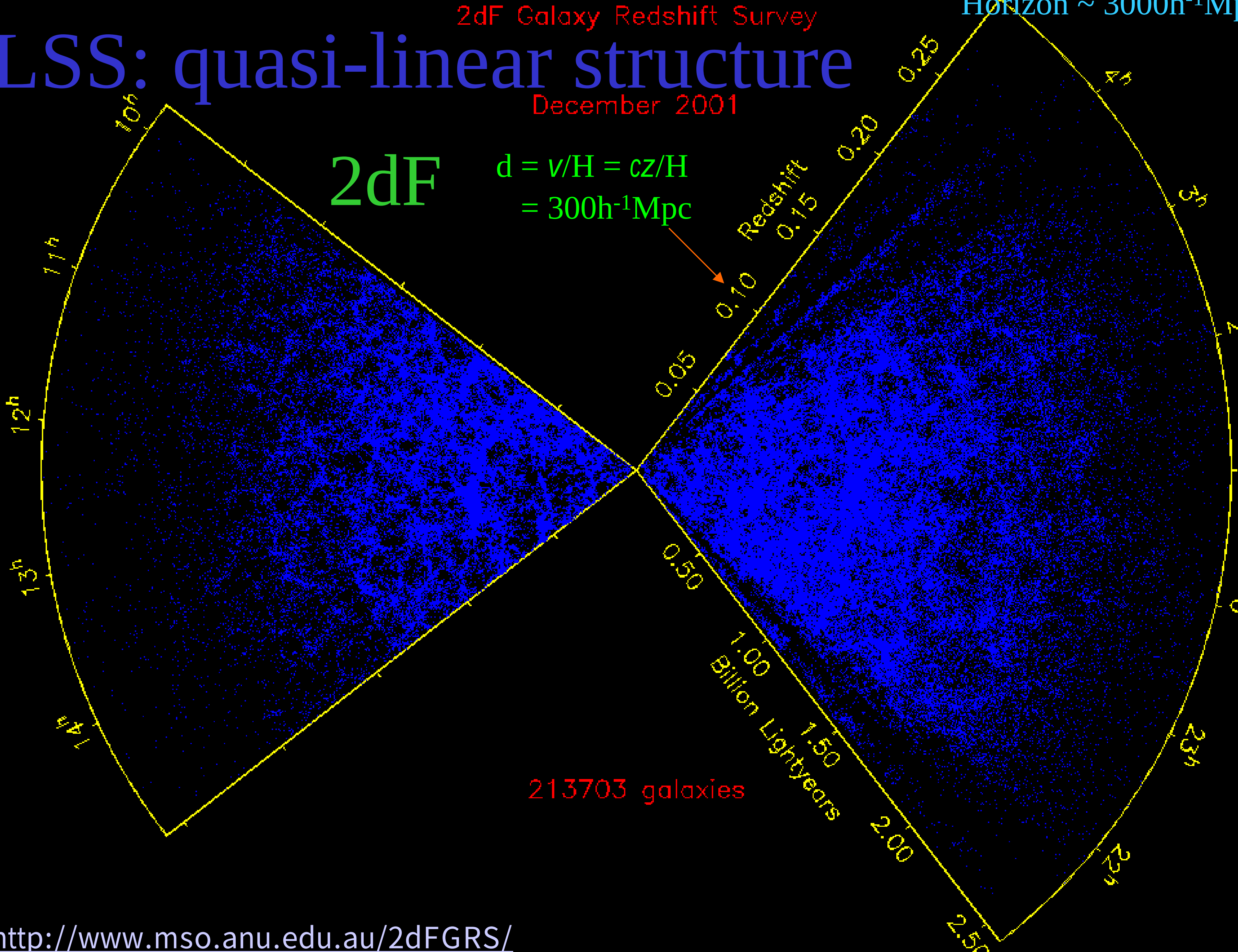
$z \sim 0.06$
$d \sim 200h^{-1}$Mpc

$z = v/c \sim 0.03$
$d = v/H \sim 100h^{-1}$Mpc

de Lapparent *etal.* (1986)

http://www.sdss.org/

LSS: quasi-linear structure



2dF

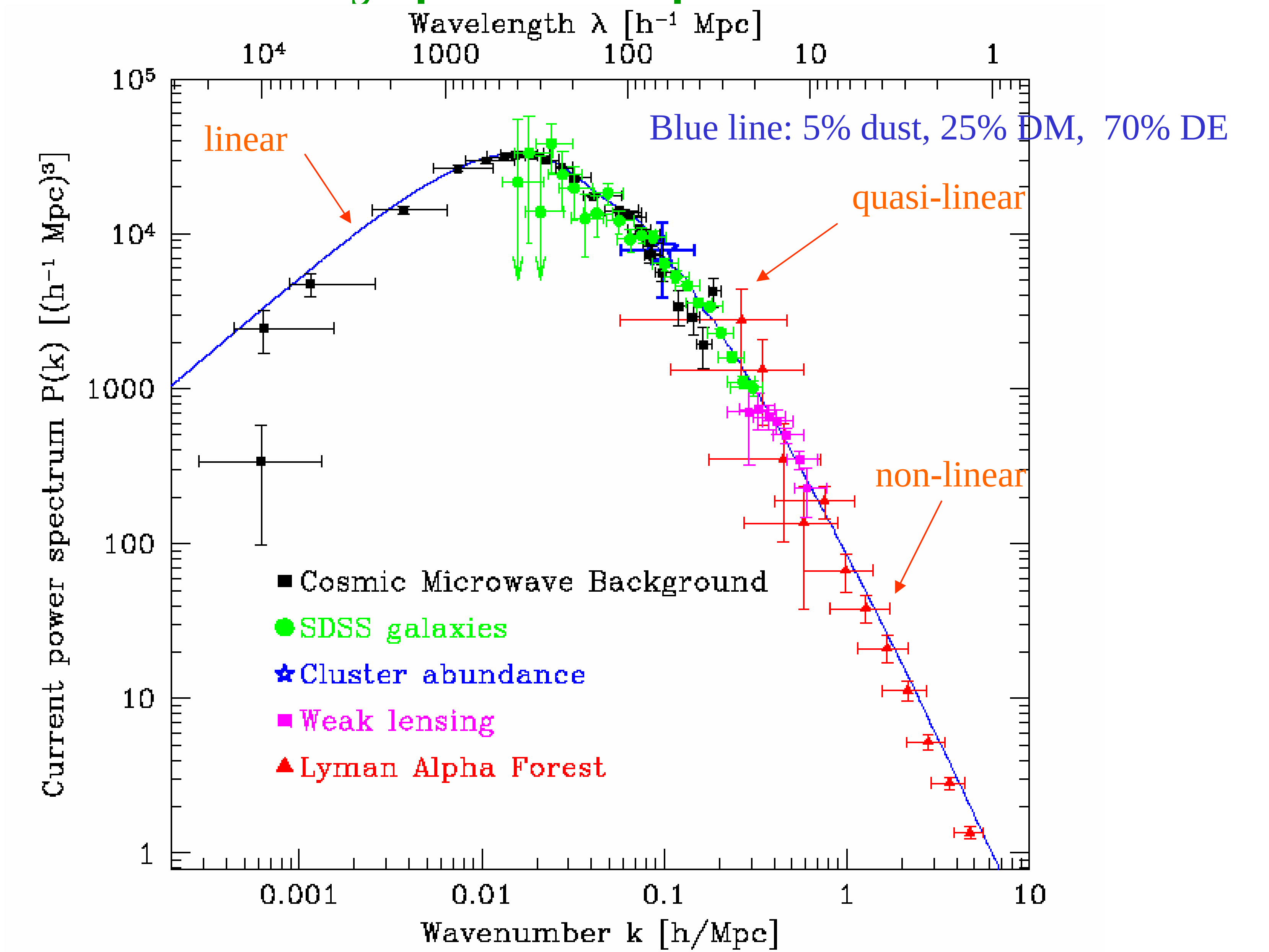
$d = v/H = cz/H$

$= 300h^{-1}$Mpc

Horizon ~ $3000h^{-1}$Mpc

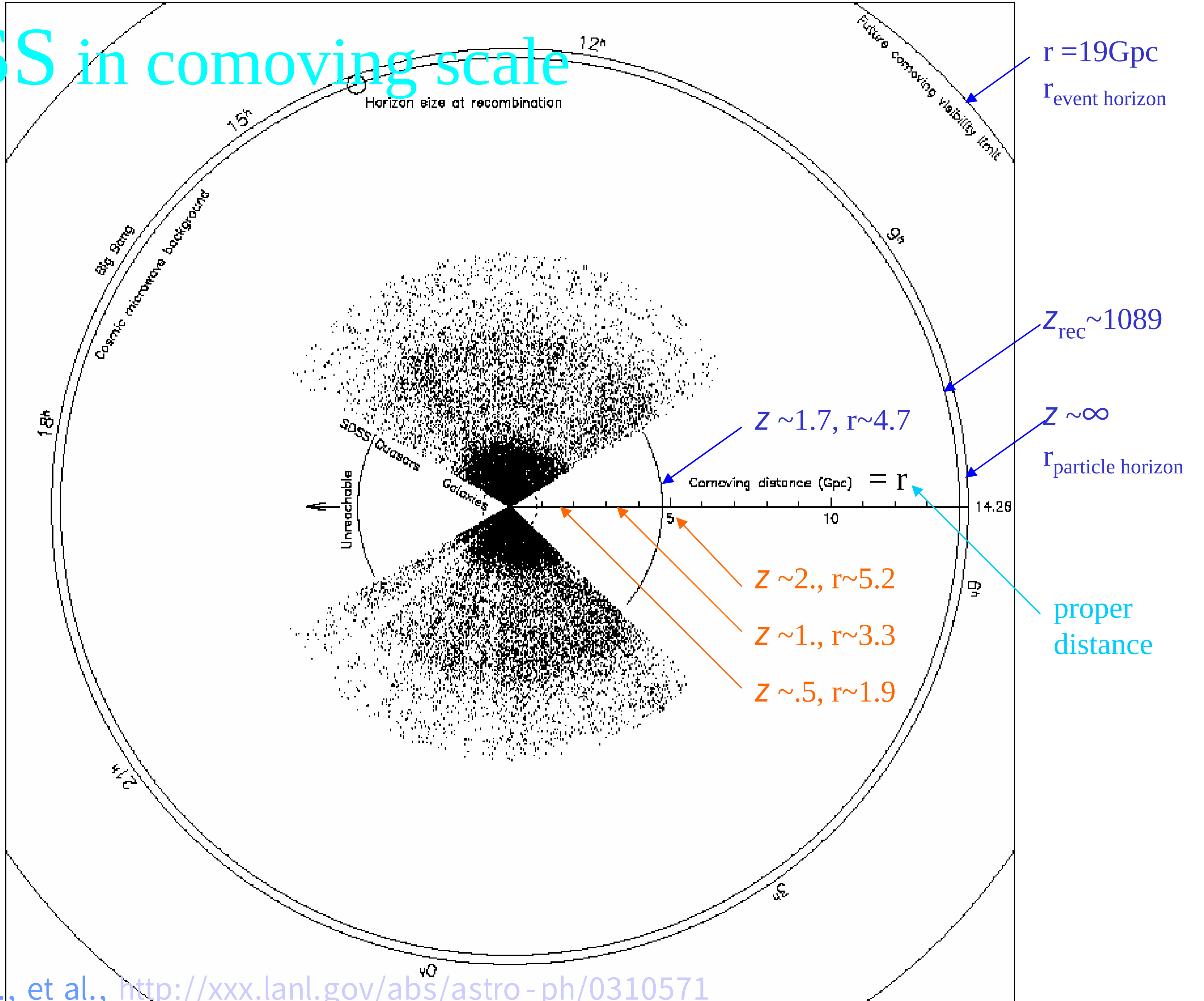
http://www.mso.anu.edu.au/2dFGRS/

Density power spectrum



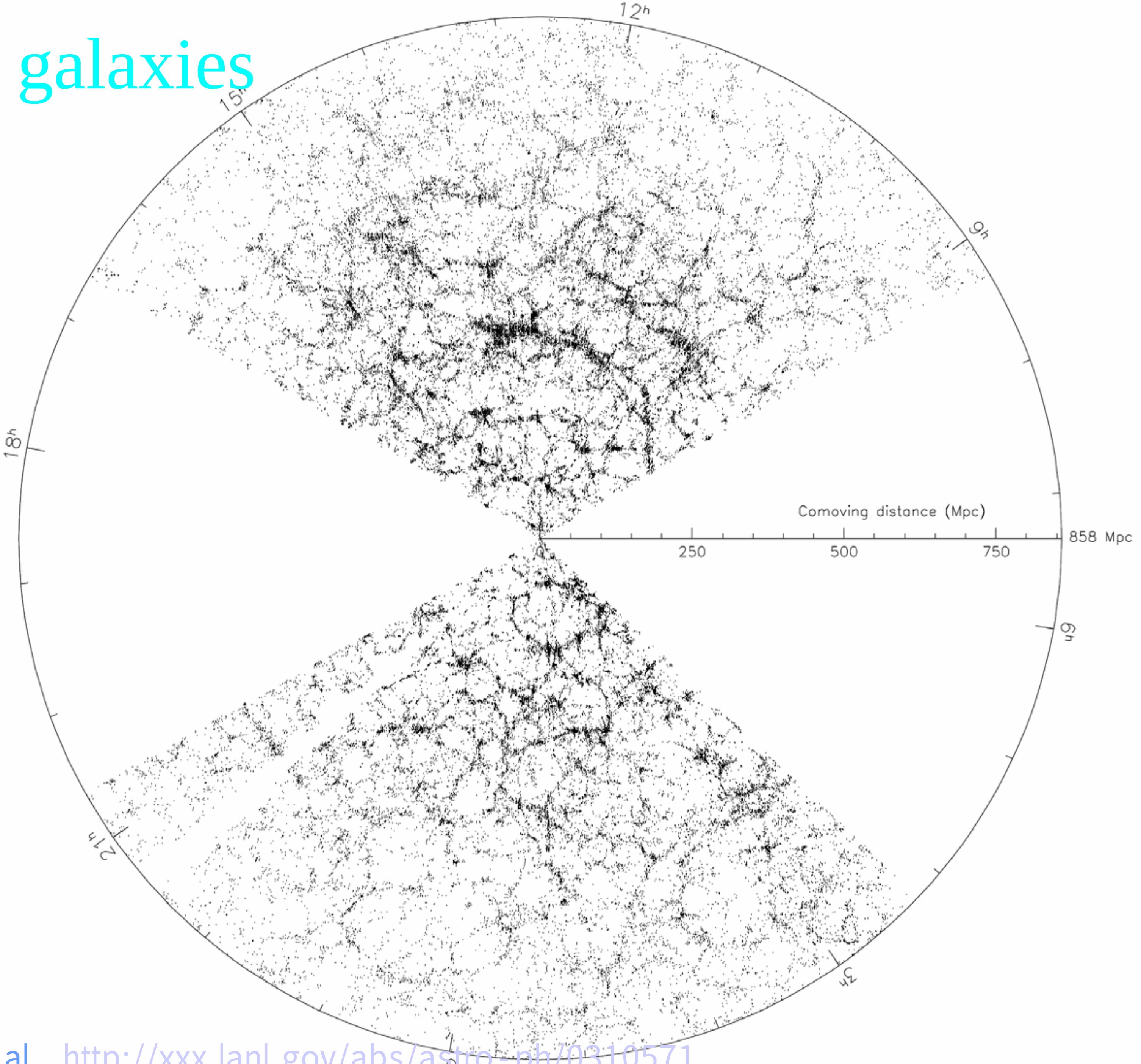
Tegmark, M., et al, http://xxx.lanl.gov/pdf/astro-ph/0207047

SDSS in comoving scale



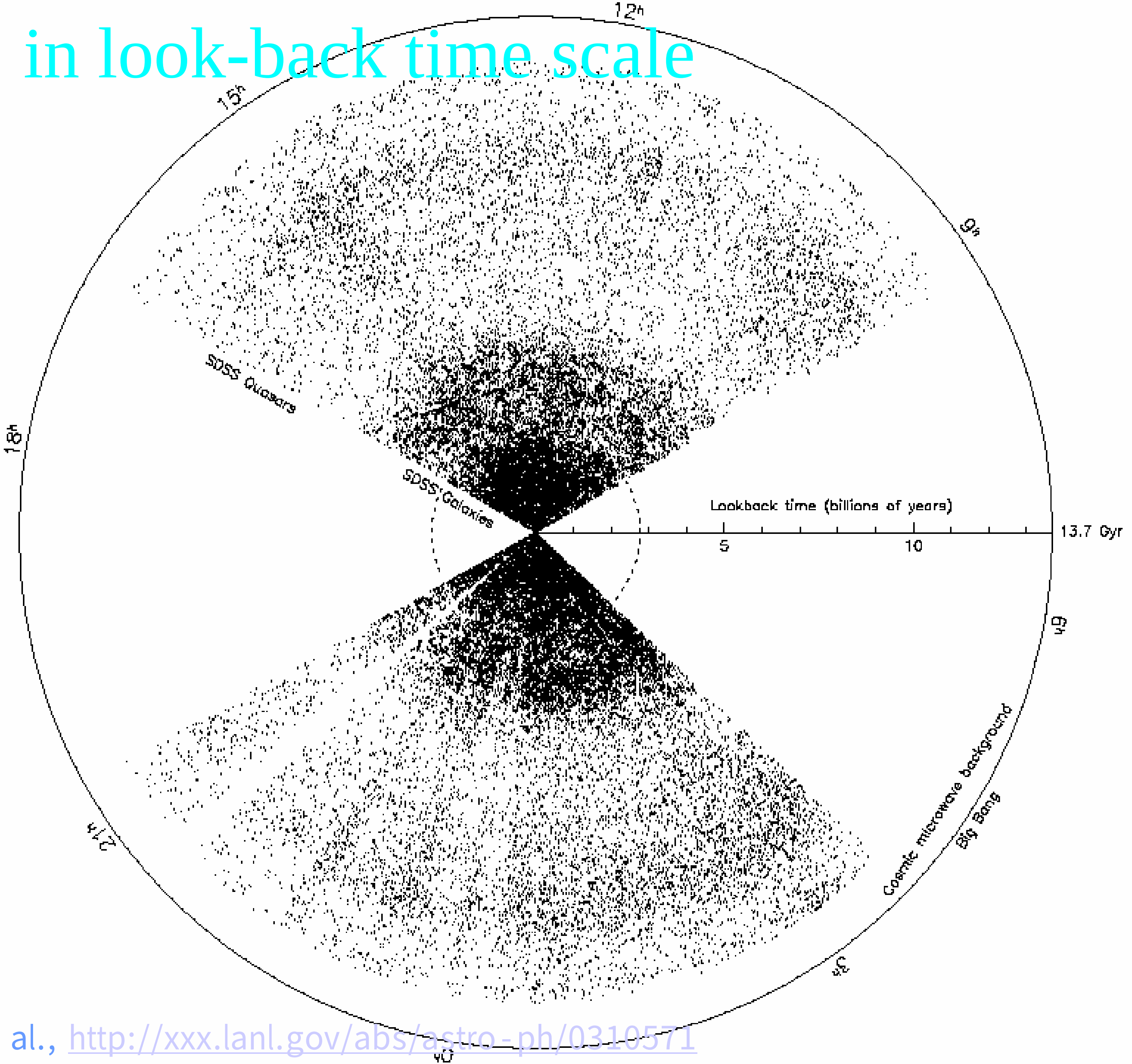
Gott, J. R., et al., http://xxx.lanl.gov/abs/astro-ph/0310571

SDSS galaxies



Gott, J. R., et al., http://xxx.lanl.gov/abs/astro-ph/0310571

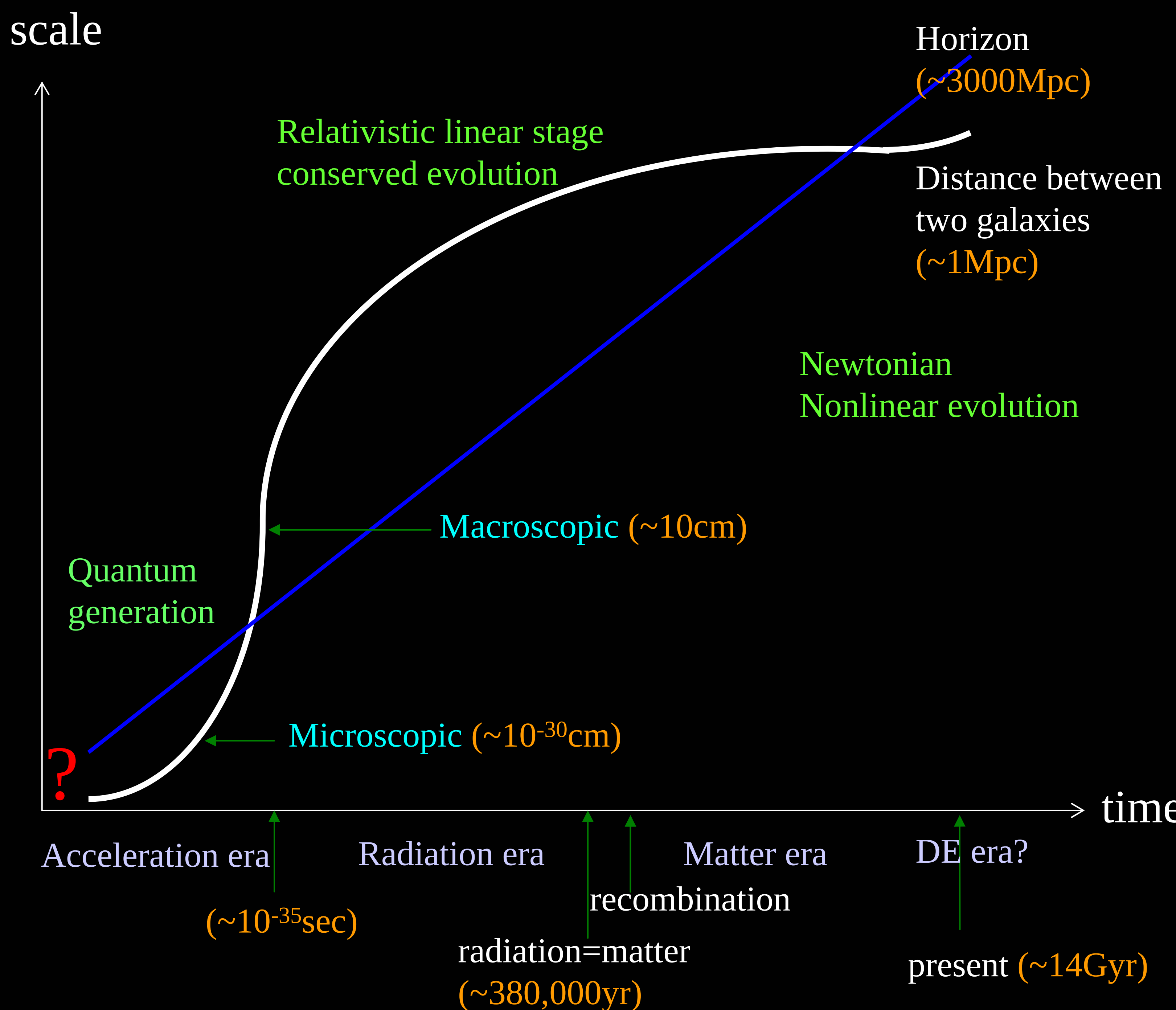
SDSS in look-back time scale



Gott, J. R., et al., http://xxx.lanl.gov/abs/astro-ph/0310571

Origin and evolution of LSS

- Quantum origin
  - Space-time quantum fluctuations from uncertainty pr.
  - Become macroscopic due to inflation.
- Linear evolution (Relativistic)
  - Linear evolution of the macroscopic seeds.
  - Structures are described by conserved amplitudes.
- Nonlinear evolution (Newtonian)
  - Nonlinear evolution inside the horizon.
  - Newtonian numerical computer simulation.



scale
Horizon
(~3000Mpc)
Relativistic linear stage
conserved evolution
Distance between
two galaxies
(~1Mpc)
Newtonian
Nonlinear evolution
Macroscopic (~10cm)
Quantum
generation
Microscopic (~10-30cm)
?
time
Acceleration era
Radiation era
Matter era
DE era?
recombination
(~10-35sec)
radiation=matter
(~380,000yr)
present (~14Gyr)

- Background world model:

  Relativistic: Friedmann (1922)

  Newtonian: Milne-McCrea (1934)

  Coincide for zero-pressure

- Linear structures:

  Relativistic: Lifshitz (1946)

  Newtonian: Bonnor (1957)

  Coincide for zero-pressure

- Second-order structures:

  Newtonian: Peebles (1980)

  Relativistic: Noh-JH (2004)

  Coincide for zero-pressure, no-rotation

- Third-order structures: Relativistic: JH-Noh (2005)

  Pure general relativistic corrections

  $\delta T/T \sim 10^{-5}$ order higher, independent of horizon

PHYSICAL REVIEW D **69**, 104011 (2004)

Second-order perturbations of the Friedmann world model

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$$\ddot{\delta}_v + 2H\dot{\delta}_v - 4\pi G\bar{\mu}\delta_v = -\frac{1}{a^2}[a\nabla\cdot(\delta_v\mathbf{u})]^{\cdot} + \frac{1}{a^2}\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u}) + \dot{C}^{(t)}_{\alpha\beta}\left(\frac{2}{a}\nabla^\alpha u^\beta + \dot{C}^{(t)\alpha\beta}\right). \quad (342)$$

Physical Review D, 69, 104011 (2004).

Newtonian equations: Peebles (1980) Fully nonlinear!

$$\dot{\delta} + \frac{1}{a}\nabla \cdot \mathbf{u} = -\frac{1}{a}\nabla \cdot (\delta \mathbf{u}), \tag{344}$$

$$\dot{\mathbf{u}} + H\mathbf{u} + \frac{1}{a}\nabla \delta\Phi = -\frac{1}{a}\mathbf{u} \cdot \nabla \mathbf{u}, \tag{345}$$

$$\frac{1}{a^2}\nabla^2 \delta\Phi = 4\pi G \bar{\varrho} \delta. \tag{346}$$

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G \bar{\varrho} \delta = -\frac{1}{a^2}[a\nabla \cdot (\delta \mathbf{u})]^{\cdot} + \frac{1}{a^2}\nabla \cdot (\mathbf{u} \cdot \nabla \mathbf{u}). \tag{343}$$

Relativistic equations: Noh-JH (2004) To second order!

$$\ddot{\delta}_v + 2H\dot{\delta}_v - 4\pi G \bar{\mu} \delta_v = -\frac{1}{a^2}[a\nabla \cdot (\delta_v \mathbf{u})]^{\cdot} + \frac{1}{a^2}\nabla \cdot (\mathbf{u} \cdot \nabla \mathbf{u}) + \dot{C}^{(t)}_{\alpha\beta}\left(\frac{2}{a}\nabla^\alpha u^\beta + \dot{C}^{(t)\alpha\beta}\right). \tag{342}$$

Physical Review D, 69, 104011 (2004).

Class. Quantum Grav. **22** (2005) 3181–3188 doi:10.1088/0264-9381/22/16/004

Relativistic-Newtonian correspondence of the zero pressure but weakly nonlinear cosmology

Hyerim Noh[1] and Jai-chan Hwang[2]

PHYSICAL REVIEW D **72,** 044011 (2005)

Second-order perturbations of a zero-pressure cosmological medium: Proofs of the relativistic-Newtonian correspondence

Jai-chan Hwang[1] and Hyerim Noh[2]

Classical and Quantum Gravity, 22, 3181 (2005); Physical Review D, 72, 044011 (2005).

Relativistic/Newtonian correspondence:

Background order:

Spatial curvature/ Total energy

Cosmological constant

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\varrho - \frac{\text{const}}{a^2} + \frac{\Lambda c^2}{3},$$

Friedmann (1922)/Milne and McCrea (1934)

Linear perturbation:

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\varrho\delta = 0,$$

Lifshitz (1946)/Bonnor (1957)

Classical and Quantum Gravity, 22, 3181 (2005).

Covariant equations: Ehlers (1961), Ellis (1971)

Energy conservation:

Energy frame

$$\dot{\tilde{\mu}}+(\tilde{\mu}+\tilde{p})\tilde{\theta}+\tilde{\pi}^{ab}\tilde{\sigma}_{ab}+\tilde{q}^{a}{}_{;a}+\tilde{q}^{a}\tilde{a}_{a}=0,$$

Zero-pressure

Raychaudhury equation:

$$\dot{\tilde{\theta}}+\frac{1}{3}\tilde{\theta}^{2}-\tilde{a}^{a}{}_{;a}+2(\tilde{\sigma}^{2}-\tilde{\omega}^{2})+4\pi G(\tilde{\mu}+3\tilde{p})-\Lambda=0.$$

Irrotational

Physical Review D, 69, 104011 (2004).

Zero-pressure fluid:

$$\dot{\tilde{\tilde{\mu}}} + \tilde{\mu}\tilde{\theta} = 0,$$

$$\dot{\tilde{\tilde{\theta}}} + \frac{1}{3}\tilde{\theta}^2 + \tilde{\sigma}^{ab}\tilde{\sigma}_{ab} - \tilde{\omega}^{ab}\tilde{\omega}_{ab} + 4\pi G\tilde{\mu} - \Lambda = 0,$$

$$\left(\frac{\dot{\tilde{\tilde{\mu}}}}{\tilde{\mu}}\right)^{\dot{\tilde{}}} - \frac{1}{3}\left(\frac{\dot{\tilde{\tilde{\mu}}}}{\tilde{\mu}}\right)^2 - \tilde{\sigma}^{ab}\tilde{\sigma}_{ab} + \tilde{\omega}^{ab}\tilde{\omega}_{ab} - 4\pi G\tilde{\mu} + \Lambda = 0.$$

Classical and Quantum Gravity, 22, 3181 (2005).

Zero-pressure, no-rotation:

Perturbed order

$$\tilde{\dot{\mu}} + \tilde{\mu}\tilde{\theta} = 0, \quad \tilde{\dot{\theta}} + \frac{1}{3}\tilde{\theta}^2 + \tilde{\sigma}^{ab}\tilde{\sigma}_{ab} + 4\pi G\tilde{\mu} - \Lambda = 0,$$

Friedmann background:

$$\tilde{\dot{\mu}} = \dot{\mu}, \quad \tilde{\theta} = 3\frac{\dot{a}}{a}$$

$$\dot{\mu} + 3\frac{\dot{a}}{a}\mu = 0, \quad 3\frac{\ddot{a}}{a} + 4\pi G\mu - \Lambda = 0.$$

General Relativity and Gravitation, 38, 703 (2006).

Zero-pressure, no-rotation, temporal comoving gauge:

$$\tilde{T}^0_0 = -\tilde{\mu}, \quad \tilde{T}^0_\alpha = 0 = \tilde{T}^\alpha_\beta,$$

Zero - pressure

Temporal comoving gauge + Irrotational

Monthly Notices of the Royal Astronomical Society, 367, 1515 (2006).

$$\dot{\tilde{\mu}} + \tilde{\mu}\tilde{\theta} = 0, \quad \dot{\tilde{\theta}} + \frac{1}{3}\tilde{\theta}^2 + \tilde{\sigma}^{ab}\tilde{\sigma}_{ab} + 4\pi G\tilde{\mu} - \Lambda = 0,$$

To linear-order perturbations:

$$\tilde{\mu} \equiv \mu + \delta\mu, \qquad \tilde{\theta} \equiv 3\frac{\dot{a}}{a} + \delta\theta,$$

$$\delta\mu \equiv \delta\varrho, \qquad \delta\theta \equiv \frac{1}{a}\nabla \cdot \mathbf{u},$$

$$\dot{\delta} + \frac{1}{a}\nabla \cdot \boldsymbol{u} = 0,$$

$$\frac{1}{a}\nabla \cdot \left(\dot{\boldsymbol{u}} + \frac{\dot{a}}{a}\boldsymbol{u}\right) + 4\pi G\mu\delta = 0.$$

Monthly Notices of the Royal Astronomical Society, 367, 1515 (2006).

$$\dot{\delta} + \frac{1}{a}\nabla \cdot \boldsymbol{u} = 0,$$

$$\frac{1}{a}\nabla \cdot \left(\dot{\boldsymbol{u}} + \frac{\dot{a}}{a}\boldsymbol{u} \right) + 4\pi G\mu\delta = 0.$$

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\mu\delta = 0,$$

Lifshitz (1946): synchronous gauge

Nariai (1969): comoving gauge

Nonlinear order

$$\left(\frac{\dot{\tilde{\mu}}}{\tilde{\mu}}\right)^{\tilde{\cdot}} - \frac{1}{3}\left(\frac{\dot{\tilde{\mu}}}{\tilde{\mu}}\right)^{2} - \tilde{\sigma}^{ab}\tilde{\sigma}_{ab} - 4\pi G\tilde{\mu} + \Lambda = 0.$$

Monthly Notices of the Royal Astronomical Society, 367, 1515 (2006).

$$\dot{\tilde{\mu}} + \tilde{\mu}\tilde{\theta} = 0, \quad \dot{\tilde{\theta}} + \frac{1}{3}\tilde{\theta}^2 + \tilde{\sigma}^{ab}\tilde{\sigma}_{ab} + 4\pi G\tilde{\mu} - \Lambda = 0,$$

To second-order perturbations:

$$\tilde{\mu} \equiv \mu + \delta\mu, \qquad \tilde{\theta} \equiv 3\frac{\dot{a}}{a} + \delta\theta,$$

$$\delta\mu \equiv \delta\varrho, \qquad \delta\theta \equiv \frac{1}{a}\nabla\cdot\mathbf{u},$$

$$\dot{\delta} + \frac{1}{a}\nabla\cdot\mathbf{u} = -\frac{1}{a}\nabla\cdot(\delta\mathbf{u}),$$

$$\frac{1}{a}\nabla\cdot\left(\dot{\mathbf{u}} + \frac{\dot{a}}{a}\mathbf{u}\right) + 4\pi G\mu\delta = -\frac{1}{a^2}\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u}) - \dot{C}^{(t)\alpha\beta}\left(\frac{2}{a}\nabla_\alpha u_\beta + \dot{C}^{(t)}_{\alpha\beta}\right),$$

Classical and Quantum Gravity, 22, 3181 (2005).

$$\ddot{\delta}_v + 2\frac{\dot{a}}{a}\dot{\delta}_v - 4\pi G\mu\delta_v = -\frac{1}{a^2}\frac{\partial}{\partial t}[a\nabla\cdot(\delta_v\mathbf{u})] + \frac{1}{a^2}\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u}) + \dot{C}^{(t)\alpha\beta}\left(\frac{2}{a}u_{\alpha,\beta} + \dot{C}^{(t)}_{\alpha\beta}\right).$$

$$\left(\frac{\dot{\tilde{\mu}}}{\tilde{\mu}}\right)^{\tilde{\cdot}} - \frac{1}{3}\left(\frac{\dot{\tilde{\mu}}}{\tilde{\mu}}\right)^2 - \tilde{\sigma}^{ab}\tilde{\sigma}_{ab} - 4\pi G\tilde{\mu} + \Lambda = 0.$$

Physical Review D, 72, 044011 (2005).

PHYSICAL REVIEW D **72**, 044012 (2005)

Third-order perturbations of a zero-pressure cosmological medium: Pure general relativistic nonlinear effects

Jai-chan Hwang[1] and Hyerim Noh[2]

Physical Review D, 72, 044012 (2005).

Linear order:

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\mu\delta = 0,$$

Second order:

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\mu\delta = -\frac{1}{a^2}\frac{\partial}{\partial t}\left[a\nabla\cdot(\delta\mathbf{u})\right] + \frac{1}{a^2}\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u}),$$

Third order:

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\mu\delta = -\frac{1}{a^2}\frac{\partial}{\partial t}\left[a\nabla\cdot(\delta\mathbf{u})\right] + \frac{1}{a^2}\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u})$$

$$+ \frac{1}{a^2}\frac{\partial}{\partial t}\{a[2\varphi\mathbf{u} - \nabla(\Delta^{-1}X)]\cdot\nabla\delta\} - \frac{4}{a^2}\nabla\cdot\left[\varphi\left(\mathbf{u}\cdot\nabla\mathbf{u} - \frac{1}{3}\mathbf{u}\nabla\cdot\mathbf{u}\right)\right]$$

$$+ \frac{2}{3a^2}\varphi\mathbf{u}\cdot\nabla(\nabla\cdot\mathbf{u}) + \frac{\Delta}{a^2}[\mathbf{u}\cdot\nabla(\Delta^{-1}X)] - \frac{1}{a^2}\mathbf{u}\cdot\nabla X - \frac{2}{3a^2}X\nabla\cdot\mathbf{u},$$

$$X \equiv 2\varphi\nabla\cdot\mathbf{u} - \mathbf{u}\cdot\nabla\varphi + \frac{3}{2}\Delta^{-1}\nabla\cdot[\mathbf{u}\cdot\nabla(\nabla\varphi) + \mathbf{u}\Delta\varphi].$$

General Relativity and Gravitation, 38, 703 (2006).

φ

$$ds^2 = -a^2(1+2\alpha)d\eta^2 - 2a^2\beta_{,\alpha}d\eta dx^\alpha + a^2[g^{(3)}_{\alpha\beta}(1+2\varphi) + 2\gamma_{,\alpha|\beta} + 2C^{(t)}_{\alpha\beta}]dx^\alpha dx^\beta,$$

To linear order:

$$R^{(h)} = \frac{6\bar{K}}{a^2} - 4\frac{\Delta + 3\bar{K}}{a^2}\varphi,$$

Curvature perturbation

In the comoving gauge, flat background:

$$\dot{\varphi}_v = 0. \qquad \varphi_v = C,$$

CMB:

Sachs-Wolfe effect

COBE, WMAP

$$\frac{\delta T}{T} \sim \frac{1}{3}\varphi_\chi = \frac{1}{3}\frac{\delta\Phi}{c^2} \sim \frac{1}{5}\varphi_v \sim \frac{1}{5}C \sim 10^{-5}$$

$$\varphi_v \sim 5 \times 10^{-5},$$

Physical Review D, 72, 044012 (2005); http://xxx.lanl.gov/abs/0704.2086

Mon. Not. R. Astron. Soc. **367,** 1515–1520 (2006) doi:10.1111/j.1365-2966.2006.10067.x

Why Newtonian gravity is reliable in large-scale cosmological simulations

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[2]*Korean Astronomy and Space Science Institute, Taejon, Korea*

1. Relativistic/Newtonian correspondence to the second order
2. Pure general relativistic third-order corrections are small $\sim 5\text{x}10^{-5}$
3. Correction terms are independent of presence of the horizon.

Monthly Notices of the Royal Astronomical Society, 367, 1515 (2006).

Assumptions:

Our relativistic/Newtonian correspondence includes Λ, but assumes:

1. Flat Friedmann background
2. Zero-pressure
3. Irrotational
4. Single component fluid
5. No gravitational waves
6. Second order in perturbations

Relaxing any of these assumptions could lead to pure general relativistic effects!

Second-order perturbations of cosmological fluids: Relativistic effects of pressure, multi-component, curvature, and rotation

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http://xxx.lanl.gov/abs/0704.1927

Effects of pressure:

To second order:

Pure relativistic corrections

$$\dot{\delta} - 3wH\delta + (1 + w)\frac{1}{a}\nabla\cdot\mathbf{u} = -\frac{1}{a}\nabla\cdot(\delta\mathbf{u}) + \frac{3}{2}\frac{c_s^2}{1+w}H\delta^2 + \delta_\Pi, \tag{123}$$

$$\frac{1}{a}\nabla\cdot(\dot{\mathbf{u}} + H\mathbf{u}) + \frac{4\pi G\mu}{c^2}\delta + \frac{c_s^2}{1+w}c^2\frac{\Delta}{a^2}\delta = -\frac{1}{a^2}\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u}) - \dot{C}^{(t)\alpha\beta}\left(\frac{2}{a}u_{\alpha|\beta} + \dot{C}^{(t)}_{\alpha\beta}\right)$$
$$+\frac{c_s^2}{1+w}\left\{\frac{1}{2}\left(-\frac{4\pi G\mu}{c^2} + \frac{1+2c_s^2}{1+w}c^2\frac{\Delta}{a^2}\right)\delta^2 + 2H\delta\frac{1}{a}\nabla\cdot\mathbf{u} + \frac{c^2}{a^2}\left[2\varphi\Delta\delta - (\nabla\varphi)\cdot\nabla\delta + 2\delta^{,\alpha|\beta}C^{(t)}_{\alpha\beta}\right]\right\} - \kappa_\Pi. \tag{124}$$

$$\frac{1+w}{a^2H}\left[\frac{H^2}{(\mu+p)a}\left(\frac{a^3\mu}{H}\delta\right)^{\cdot}\right]^{\cdot} - c_s^2c^2\frac{\Delta}{a^2}\delta$$
$$= \frac{1+w}{a^2}\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u}) - \frac{1+w}{a^2}\left\{\frac{a}{1+w}\left[\nabla\cdot(\delta\mathbf{u}) - \frac{3}{2}aH\frac{c_s^2}{1+w}\delta^2\right]\right\}^{\cdot} + (1+w)\dot{C}^{(t)\alpha\beta}\left(\frac{2}{a}u_{\alpha|\beta} + \dot{C}^{(t)}_{\alpha\beta}\right)$$
$$+\frac{1}{2}c_s^2\left(\frac{4\pi G\mu}{c^2} - \frac{1+2c_s^2}{1+w}c^2\frac{\Delta}{a^2}\right)\delta^2 - c_s^2\frac{1}{a^2}\left[2aH\delta\nabla\cdot\mathbf{u} + 2\varphi c^2\Delta\delta - c^2(\nabla\varphi)\cdot\nabla\delta + 2c^2\delta^{,\alpha|\beta}C^{(t)}_{\alpha\beta}\right]$$
$$+(1+w)\kappa_\Pi + \frac{1+w}{a^2}\left(\frac{a^2}{1+w}\delta_\Pi\right)^{\cdot} - \frac{1}{a}\mathbf{u}\cdot\nabla\delta_\Pi. \tag{125}$$

http://xxx.lanl.gov/abs/0704.1927

To background order:

$$H^2 = \frac{8\pi G}{3}\mu - \frac{K}{a^2} + \frac{\Lambda}{3},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\mu + 3p) + \frac{\Lambda}{3},$$

$$\dot{\mu} + 3H(\mu + p) = 0,$$

Pure relativistic corrections

To linear order:

$$\dot{\delta} - 3wH\delta + (1 + w)\frac{1}{a}\nabla \cdot \mathbf{u} = 0,$$

$$\frac{1}{a}\nabla \cdot (\dot{\mathbf{u}} + H\mathbf{u}) + 4\pi G\varrho\delta = -\frac{1}{1+w}\frac{\Delta}{a^2}\frac{\delta p}{\varrho},$$

$$\frac{1+w}{a^2 H}\left[\frac{H^2}{a(1+w)\varrho}\left(\frac{a^3\varrho}{H}\delta\right)^{\cdot}\right]^{\cdot} = \frac{\Delta}{a^2}\frac{\delta p}{\varrho},$$

Physical Review D, 69, 104011 (2004); http://xxx.lanl.gov/abs/0704.1927

Effects of multi-component:

Newtonian equations:

Pressure

$$\dot{\delta}_i + \frac{1}{a}\nabla\cdot\mathbf{u}_i = -\frac{1}{a}\nabla\cdot(\delta_i\mathbf{u}_i)\,,$$

$$\dot{\mathbf{u}}_i + H\mathbf{u}_i + \frac{1}{a}\mathbf{u}_i\cdot\nabla\mathbf{u}_i = \boxed{-\frac{1}{a\bar{\varrho}_i}\frac{\nabla\delta p_i}{1+\delta_i}} - \frac{1}{a}\nabla\delta\Phi,$$

$$\frac{1}{a^2}\nabla^2\delta\Phi = 4\pi G\sum_j \bar{\varrho}_j\delta_j.$$

$$\ddot{\delta}_i + 2H\dot{\delta}_i - 4\pi G\sum_j\bar{\varrho}_j\delta_j = -\frac{1}{a^2}\left[a\nabla\cdot(\delta_i\mathbf{u}_i)\right]^{\cdot} + \frac{1}{a^2}\nabla\cdot(\mathbf{u}_i\cdot\nabla\mathbf{u}_i) \boxed{+\frac{1}{a^2\bar{\varrho}_i}\nabla\cdot\left(\frac{\nabla\delta p_i}{1+\delta_i}\right)}.$$

These are fully nonlinear equations!

http://xxx.lanl.gov/abs/0704.1927

Relativistic equations:

Gravitational waves

$$\dot{\delta}_i + \frac{1}{a}\nabla \cdot \mathbf{u}_i = -\frac{1}{a}\nabla \cdot (\delta_i \mathbf{u}_i)\,,$$

$$\frac{1}{a}\nabla \cdot (\dot{\mathbf{u}}_i + H\mathbf{u}_i) + 4\pi G\varrho\delta = -\frac{1}{a^2}\nabla \cdot (\mathbf{u}_i \cdot \nabla \mathbf{u}_i) - \dot{C}^{(t)\alpha\beta}\left(\frac{2}{a}u_{\alpha|\beta} + \dot{C}^{(t)}_{\alpha\beta}\right),$$

$$\frac{1}{a^2}\left(a^2\dot{\delta}_i\right)^{\cdot} - 4\pi G\varrho\delta = -\frac{1}{a^2}\left[a\nabla \cdot (\delta_i \mathbf{u}_i)\right]^{\cdot} + \frac{1}{a^2}\nabla \cdot (\mathbf{u}_i \cdot \nabla \mathbf{u}_i) + \dot{C}^{(t)}_{\alpha\beta}\left(\frac{2}{a}u_i^{\alpha|\beta} + \dot{C}^{(t)\alpha\beta}\right).$$

We have ignored pure decaying contributions:

$$\mathbf{u}_i - \mathbf{u} = \frac{1}{a}\left[\nabla d_i(\mathbf{x}) + \mathbf{D}_i(\mathbf{x}) - \mathbf{D}(\mathbf{x})\right],$$

$$\sum_j \varrho_j d_j \equiv 0, \quad \nabla \cdot \mathbf{D} \equiv 0 \equiv \nabla \cdot \mathbf{D}_i.$$

→ Relativistic/Newtonian correspondence!

http://xxx.lanl.gov/abs/0704.1927

Effects of curvature:

Pure relativistic corrections

$$\dot{\delta} + \frac{1}{a}\nabla\cdot\mathbf{u} = -\frac{1}{a}\nabla\cdot(\delta\mathbf{u}) + \frac{1}{a}(\nabla\delta)\cdot\nabla\left(\frac{3K}{\Delta+3K}u\right),$$

$$\frac{1}{a}\nabla\cdot(\dot{\mathbf{u}}+H\mathbf{u}) + 4\pi G\varrho\delta = -\frac{1}{a^2}\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u}) - \dot{C}^{(t)\alpha\beta}\left(\dot{C}^{(t)}_{\alpha\beta} + \frac{2}{a}u_{,\alpha|\beta}\right) + \frac{1}{a^2}\nabla\cdot\left[\left(\frac{3K}{\Delta+3K}\mathbf{u}\right)\cdot\nabla\mathbf{u}\right]$$
$$-\frac{1}{3}\frac{1}{a^2}\left(\frac{3K}{\Delta+3K}\nabla\cdot\mathbf{u}\right)\left(\frac{2\Delta+3K}{\Delta+3K}\nabla\cdot\mathbf{u}\right) + \frac{1}{a}\left(\frac{3K}{\Delta+3K}u\right)^{,\alpha|\beta}\left[2\dot{C}^{(t)}_{\alpha\beta} + \frac{1}{a}\left(\frac{\Delta}{\Delta+3K}u\right)_{,\alpha|\beta}\right].$$

where

$$K = \left(\frac{aH}{c}\right)^2(\Omega_t - 1), \quad \Omega_t \equiv \Omega + \Omega_\Lambda, \quad \Omega \equiv \frac{8\pi G\varrho}{3H^2}, \quad \Omega_\Lambda \equiv \frac{\Lambda c^2}{3H^2}.$$

http://xxx.lanl.gov/abs/0704.1927

Effects of rotation:

Pure relativistic corrections

$$\dot{\delta} + \frac{1}{a}\nabla\cdot\mathbf{u} = -\frac{1}{a}\nabla\cdot(\delta\mathbf{U}) + \frac{2}{a}C^{(t)\alpha\beta}u^{(v)}_{\alpha|\beta} - \frac{1}{a}\mathbf{u}^{(v)}\cdot\nabla\varphi + \frac{1}{c^2}H\left[\mathbf{u}^{(v)}\cdot\mathbf{u}^{(v)} + 3\Delta^{-1}\nabla^{\alpha}\left(u^{(v)}_{\alpha|\beta}U^{\beta} + u^{(v)\beta}\tilde{U}_{\beta|\alpha}\right)\right], \quad (247)$$

$$\frac{1}{a}\nabla\cdot(\dot{\mathbf{u}} + H\mathbf{u}) + 4\pi G\varrho\delta = -\frac{1}{a^2}\nabla\cdot(\mathbf{U}\cdot\nabla\mathbf{U}) - \dot{C}^{(t)\alpha\beta}\left(\dot{C}^{(t)}_{\alpha\beta} + \frac{2}{a}\tilde{U}_{\alpha|\beta}\right) + \frac{8\pi G\varrho}{c^2}\left[-\mathbf{u}^{(v)}\cdot\mathbf{u}^{(v)} + \frac{3}{2}\Delta^{-1}\nabla^{\alpha}\left(u^{(v)}_{\alpha|\beta}U^{\beta} + u^{(v)\beta}\tilde{U}_{\beta|\alpha}\right)\right], \quad (248)$$

$$\dot{\mathbf{u}}^{(v)} + H\mathbf{u}^{(v)} = -\frac{1}{a}\left[\mathbf{U}\cdot\nabla\mathbf{u} - \nabla\Delta^{-1}\nabla\cdot(\mathbf{U}\cdot\nabla\mathbf{u})\right] - \frac{1}{a}\left[U^{\beta}c\Psi^{(v)}_{\beta|\alpha} - \nabla_{\alpha}\Delta^{-1}\nabla^{\gamma}\left(U^{\beta}c\Psi^{(v)}_{\beta|\gamma}\right)\right]. \quad (249)$$

where

$$U_{\alpha} \equiv u_{\alpha} + c\Psi^{(v)}_{\alpha}, \quad \tilde{U}_{\alpha} \equiv u_{,\alpha} + c\Psi^{(v)}_{\alpha}. \qquad \Psi^{(v)}_{\alpha} \equiv B^{(v)}_{\alpha} + c^{-1}a\dot{C}^{(v)}_{\alpha},$$

$$A \equiv \alpha, \quad B_{\alpha} \equiv \beta_{,\alpha} + B^{(v)}_{\alpha}, \quad C_{\alpha\beta} \equiv \varphi g^{(3)}_{\alpha\beta} + \gamma_{,\alpha|\beta} + C^{(v)}_{(\alpha|\beta)} + C^{(t)}_{\alpha\beta},$$

$$\tilde{g}_{00} \equiv -a^2(1+2A), \quad \tilde{g}_{0\alpha} \equiv -a^2B_{\alpha}, \quad \tilde{g}_{\alpha\beta} \equiv a^2\left(g^{(3)}_{\alpha\beta} + 2C_{\alpha\beta}\right).$$

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Small-scale limit:

Gravitational waves

$$\dot{\delta} + \frac{1}{a}\nabla\cdot\mathbf{u} = -\frac{1}{a}\nabla\cdot(\delta\mathbf{u}) + \frac{2}{a}C^{(t)\alpha\beta}u^{(v)}_{\alpha|\beta},$$

$$\frac{1}{a}\nabla\cdot(\dot{\mathbf{u}} + H\mathbf{u}) + 4\pi G\varrho\delta = -\frac{1}{a^2}\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u}) - \dot{C}^{(t)\alpha\beta}\left[\dot{C}^{(t)}_{\alpha\beta} + \frac{2}{a}\left(u_{,\alpha|\beta} + c\Psi^{(v)}_{\alpha|\beta}\right)\right],$$

$$\dot{\mathbf{u}}^{(v)} + H\mathbf{u}^{(v)} = -\frac{1}{a}\left[\mathbf{u}\cdot\nabla\mathbf{u} - \nabla\Delta^{-1}\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u})\right].$$

➡ Relativistic/Newtonian correspondence!

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Third-order cosmological perturbations of zero-pressure multi-component fluids: Pure general relativistic nonlinear effects

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Ignoring quadratic combinations of pure decaying terms:

$$\dot{\delta} + \frac{1}{a}\nabla\cdot\mathbf{u} = -\frac{1}{a}\nabla\cdot(\delta\mathbf{u}) + \frac{1}{a}\left(2\varphi\mathbf{u} - \nabla\Delta^{-1}X\right)\cdot\nabla\delta,$$

$$\frac{1}{a}\nabla\cdot(\dot{\mathbf{u}} + H\mathbf{u}) \; + 4\pi G\varrho\delta = -\frac{1}{a^2}\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u}) - \frac{\Delta}{a^2}\left[\mathbf{u}\cdot\nabla\left(\Delta^{-1}X\right)\right] + \frac{1}{a^2}\left(\mathbf{u}\cdot\nabla X + \frac{2}{3}X\nabla\cdot\mathbf{u}\right)$$

$$-\frac{2}{3a^2}\varphi\mathbf{u}\cdot\nabla\left(\nabla\cdot\mathbf{u}\right) + \frac{4}{a^2}\nabla\cdot\left[\varphi\left(\mathbf{u}\cdot\nabla\mathbf{u} - \frac{1}{3}\mathbf{u}\nabla\cdot\mathbf{u}\right)\right],$$

$$\dot{\delta}_i + \frac{1}{a}\nabla\cdot\mathbf{u}_i = -\frac{1}{a}\nabla\cdot(\delta_i\mathbf{u}_i) + \frac{1}{a}\left[2\varphi\mathbf{u}_i - \nabla\left(\Delta^{-1}X\right)\right]\cdot\nabla\delta_i \boxed{+ \frac{1}{a}\left[2\varphi\nabla\cdot(\mathbf{u}_i - \mathbf{u}) - (\mathbf{u}_i - \mathbf{u})\cdot\nabla\varphi\right]}$$

$$\boxed{+\frac{2}{a}\varphi\left[\delta_i\nabla\cdot(\mathbf{u}_i - \mathbf{u}) + 2\left(\mathbf{u}_i - \mathbf{u}\right)\cdot\nabla\varphi - 2\varphi\nabla\cdot(\mathbf{u}_i - \mathbf{u})\right] - \frac{1}{a}\delta_i\left(\mathbf{u}_i - \mathbf{u}\right)\cdot\nabla\varphi,}$$

$$\frac{1}{a}\nabla\cdot(\dot{\mathbf{u}}_i + H\mathbf{u}_i) + 4\pi G\varrho\delta = -\frac{1}{a^2}\nabla\cdot(\mathbf{u}_i\cdot\nabla\mathbf{u}_i) - \frac{\Delta}{a^2}\left[\mathbf{u}_i\cdot\nabla\left(\Delta^{-1}X\right)\right] + \frac{1}{a^2}\left(\mathbf{u}\cdot\nabla X + \frac{2}{3}X\nabla\cdot\mathbf{u}\right)$$

$$-\frac{2}{3a^2}\varphi\mathbf{u}\cdot\nabla\left(\nabla\cdot\mathbf{u}\right) + \frac{4}{a^2}\nabla\cdot\left[\varphi\left(\mathbf{u}\cdot\nabla\mathbf{u} - \frac{1}{3}\mathbf{u}\nabla\cdot\mathbf{u}\right)\right] \boxed{+ 2\frac{\Delta}{a^2}\left[\varphi\mathbf{u}\cdot(\mathbf{u}_i - \mathbf{u})\right].}$$

Pure decaying terms

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Einstein's gravity corrections to Newtonian cosmology:

1. Relativistic/Newtonian correspondence for a zero-pressure, irrotational fluid in flat background without gravitational waves.
2. Gravitational waves → Corrections
3. Third-order perturbations → Corrections
   → Small, independent of horizon
4. Background curvature → Corrections
5. Pressure → Relativistic even to the background and linear order
6. Rotation → Corrections
   → Newtonian correspondence in the small-scale limit
7. Multi-component zero-pressure irrotational fluids
   → Newtonian correspondence
8. Multi-component, third-order perturbations → Corrections
   → Small, independent of horizon

Perturbation method:

- Perturbation expansion.
- All perturbation variables are small.
- Weakly nonlinear.
- Strong gravity; fully relativistic!
- Valid in all scales!

Post-Newtonian method:

- Abandon geometric spirit of GR: recover the good old absolute space and absolute time.
- Provide GR correction terms in the Newtonian equations of motion.
- Expansion in $\frac{GM}{Rc^2} \sim \frac{v^2}{c^2} << 1$
- Fully nonlinear!
- No strong gravity situation; weakly relativistic.
- Valid far inside horizon

Cosmological nonlinear hydrodynamics with post-Newtonian corrections

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Newtonian Limit:

$$
\begin{aligned}
&\frac{1}{a^3}\left(a^3\varrho\right)^{\cdot} + \frac{1}{a}\nabla_i\left(\varrho v^i\right) = 0,\\
&\frac{1}{a}\left(a v_i\right)^{\cdot} + \frac{1}{a} v^j \nabla_j v_i + \frac{1}{a\varrho}\left(\nabla_i p + \nabla_j \Pi_i^j\right) - \frac{1}{a}\nabla_i U = 0,\\
&\left(\frac{\partial}{\partial t} + \frac{1}{a}\mathbf{v}\cdot\nabla\right)\Pi + \left(3\frac{\dot a}{a} + \frac{1}{a}\nabla\cdot\mathbf{v}\right)\frac{p}{\varrho} + \frac{1}{\varrho a}\left(Q^i{}_{|i} + \Pi_j^i v^j{}_{|i}\right) = 0,\\
&\frac{\Delta}{a^2}U + 4\pi G\left(\varrho - \varrho_b\right) = 0.
\end{aligned}
$$

Newtonian, indeed!

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First Post-Newtonian equations:

$$\frac{1}{a^3}\left(a^3\varrho^*\right)^{\cdot} + \frac{1}{a}\left(\varrho^* v^i\right)_{|i} = 0,$$

$$\frac{1}{a}\left(a v_i^*\right)^{\cdot} + \frac{1}{a} v^*_{i|j} v^j = -\frac{1}{a}\left(1 + \frac{1}{c^2} 2U\right)\frac{p_{,i}}{\varrho^*}$$

$$+\frac{1}{a}\left[1 + \frac{1}{c^2}\left(\frac{3}{2}v^2 - U + \Pi + \frac{p}{\varrho}\right)\right] U_{,i} + \frac{1}{c^2}\frac{1}{a}\left(2\Phi_{,i} - v^j P_{j|i}\right),$$

$$\varrho^* \equiv \varrho\left[1 + \frac{1}{c^2}\left(\frac{1}{2}v^2 + 3U\right)\right], \quad v_i^* \equiv v_i + \frac{1}{c^2}\left[\left(\frac{1}{2}v^2 + 3U + \Pi + \frac{p}{\varrho}\right) v_i - P_i\right].$$

$$\boxed{\frac{\Delta}{a^2}U} + 4\pi G\left(\varrho - \varrho_b\right) + \frac{1}{c^2}\left\{\frac{1}{a^2}\left[2\Delta\Phi - 2U\Delta U + \left(a P^i{}_{|i}\right)^{\cdot}\right] + \boxed{3\ddot{U} + 9\frac{\dot{a}}{a}\dot{U} + 6\frac{\ddot{a}}{a}U}\right.$$

$$\left. + 8\pi G\left[\varrho v^2 + \frac{1}{2}\left(\varrho\Pi - \varrho_b\Pi_b\right) + \frac{3}{2}\left(p - p_b\right)\right]\right\} = 0,$$

$$\frac{\Delta}{a^2}P_i = -16\pi G\varrho v_i + \frac{1}{a}\left(\frac{1}{a}P^j{}_{|j} + 4\dot{U} + 4\frac{\dot{a}}{a}U\right)_{,i}.$$

Notice: Laplacian → d'Alembertian, depending on the gauge choice

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