Axion as a cold dark matter candidate

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A R T I C L E   I N F O

A B S T R A C T

Here we generally *prove* that the axion as a coherently oscillating scalar field acts as a cold dark matter in nearly all cosmologically relevant scales. The proof is made in the linear perturbation order. Compared with our previous proof based on solutions, here we compare the equations in the axion with the ones in the cold dark matter, thus expanding the valid range of the proof. Deviation from purely pressureless medium appears in very small scale where axion *reveals* a peculiar equation of state. Our analysis is made in the presence of the cosmological constant, and our conclusions are valid in the presence of other fluid and field components.

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Cold dark matter (CDM), despite its unknown nature, became an essential ingredient in the current cosmological studies concerning the large-scale structure formation. CDM in a cosmological constant dominated world model (often termed the ΛCDM model) is currently the most successful candidate of cosmological models. The nature of CDM and the nature of cosmological constant (or some other dynamical dark energy), however, still remain as fundamental mysteries of present day physical cosmology. From early days of dark matter studies axion as a coherently oscillating scalar field is widely accepted as a candidate for the CDM [1]. Confirmation of the case using the relativistic linear perturbation analysis was made previously [2,3], for a recent study see [4]. In this work we present a more general proof that the axion can be regarded as the CDM to the linear order perturbation. We also derive an effective equation of state of the axion which could be important in the solar system scale if the system is in the linear regime. We set $c \equiv 1 \equiv \hbar$.

We consider the axion as a minimally coupled scalar field with $V = \frac{1}{2}m^2\phi^2$. The relevant current scales we are concerned correspond to

$$\frac{H}{m} = 2.133 \times 10^{-28} h \left(\frac{m}{10^{-5}\,\text{eV}} \right)^{-1}, \tag{1}$$

where we set currently $H \equiv 100h$ km/(s Mpc). In the following analyses we strictly ignore H/m higher order terms. But we do not impose any limit on the wavenumber k; this is more general than our previous work in [3] and [4]. We *take* a spatially flat background with the cosmological constant Λ and the axion; inclusion of Λ is also more general than previous studies.

We consider the temporal average of the oscillating scalar field contributes to the background fluid quantities, thus

$$H^2 = \frac{8\pi G}{3}\mu + \frac{\Lambda}{3}, \qquad \dot{H} = -4\pi G(\mu + p),$$

$$\mu = \frac{1}{2}\langle \dot{\phi}^2 + m^2\phi^2 \rangle, \qquad p = \frac{1}{2}\langle \dot{\phi}^2 - m^2\phi^2 \rangle, \tag{2}$$

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0, \tag{3}$$

where the angular bracket indicates averaging over time scale of order m^{-1}, see Eq. (5) in [3]. Under an *ansatz*

$$\phi(t) = \phi_+(t)\sin(mt) + \phi_-(t)\cos(mt), \tag{4}$$

ignoring H/m higher order terms, Eq. (3) leads to an approximate solution [2]

$$\phi(t) = a^{-3/2}[\phi_{+0}\sin(mt) + \phi_{-0}\cos(mt)], \tag{5}$$

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where ϕ_{+0} and ϕ_{-0} are constant coefficients. Thus,

$$\mu = \frac{1}{2}m^2 a^{-3}(\phi_{+0}^2 + \phi_{-0}^2), \qquad p = 0, \tag{6}$$

and the background medium evolves exactly same as a pressure-less ideal fluid [5].

We consider only the scalar-type perturbations. Our conventions are [6]

$$ds^2 = -(1 + 2\alpha)\,dt^2 - 2a\beta_{,\alpha}\,dt\,dx^\alpha$$
$$+ a^2\big[(1 + 2\varphi)\delta_{\alpha\beta} + 2\gamma_{,\alpha\beta}\big]dx^\alpha\,dx^\beta, \tag{7}$$

$$T_0^0 = -\mu - \delta\mu, \qquad T_\alpha^0 = -\frac{1}{k}(\mu + p)v_{,\alpha},$$

$$T_\beta^\alpha = (p + \delta p)\delta_\beta^\alpha + \frac{1}{a^2}\left(\nabla^\alpha\nabla_\beta - \frac{1}{3}\Delta\delta_\beta^\alpha\right)\sigma. \tag{8}$$

To the linear order, both the vector-type (rotation) and the tensor-type (gravitational waves) perturbation equations are not directly affected by the presence of the minimally coupled scalar field including the axion [6]. The basic perturbation equations we need are the Raychaudhury equation, the energy-conservation equation, and the momentum conservation equation, respectively [6]

$$\dot{\kappa} + 2H\kappa + \left(3\dot{H} - \frac{k^2}{a^2}\right)\alpha = 4\pi G(\delta\mu + 3\delta p), \tag{9}$$

$$\delta\dot{\mu} + 3H(\delta\mu + \delta p) = (\mu + p)\left(\kappa - 3H\alpha - \frac{k}{a}v\right), \tag{10}$$

$$\frac{[a^4(\mu + p)v]^{\cdot}}{a^4(\mu + p)} = \frac{k}{a}\alpha + \frac{k}{a(\mu + p)}\left(\delta p - \frac{2}{3}\frac{k^2}{a^2}\sigma\right). \tag{11}$$

The temporal average of oscillating scalar field contributes to perturbed fluid quantities as [6]

$$\delta\mu = \langle\dot{\phi}\delta\dot{\phi} - \dot{\phi}^2\alpha + m^2\phi\delta\phi\rangle,$$
$$\delta p = \langle\dot{\phi}\delta\dot{\phi} - \dot{\phi}^2\alpha - m^2\phi\delta\phi\rangle,$$
$$\frac{a}{k}(\mu + p)v = \langle\dot{\phi}\delta\phi\rangle, \qquad \sigma = 0. \tag{12}$$

For the scalar field we have the equation of motion [6]

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} + \frac{k^2}{a^2}\delta\phi + V_{,\phi\phi}\delta\phi$$
$$= \dot{\phi}(\kappa + \dot{\alpha}) + (2\ddot{\phi} + 3H\dot{\phi})\alpha. \tag{13}$$

The above set of equations is spatially gauge-invariant. But we have not taken the temporal gauge (hypersurface) condition which will be used as an advantage in handling the mathematical analyses.

For $\delta\phi$ we take an *ansatz*

$$\delta\phi(k, t) = \delta\phi_+(k, t)\sin(mt) + \delta\phi_-(k, t)\cos(mt). \tag{14}$$

Using Eqs. (5) and (14), Eq. (12) gives to the leading order in H/m

$$\delta\mu = a^{-3/2}m\bigg[m(\phi_{+0}\delta\phi_+ + \phi_{-0}\delta\phi_-)$$
$$+ \frac{1}{2}(\phi_{+0}\delta\dot{\phi}_- - \phi_{-0}\delta\dot{\phi}_+)\bigg] - \mu\alpha,$$

$$\delta p = \frac{1}{2}a^{-3/2}m(\phi_{+0}\delta\dot{\phi}_- - \phi_{-0}\delta\dot{\phi}_+) - \mu\alpha,$$

$$\frac{a}{k}(\mu + p)v = \frac{1}{2}a^{-3/2}m(\phi_{+0}\delta\phi_- - \phi_{-0}\delta\phi_+). \tag{15}$$

We have not taken the temporal gauge condition yet.

The comoving gauge takes $v = 0$ as the temporal gauge (hypersurface) condition; in the presence of additional fluid, like dust, radiation, neutrinos, etc., our gauge corresponds to the axion-comoving gauge. Eq. (11) gives

$$\alpha = -\frac{\delta p}{\mu}. \tag{16}$$

Eq. (10), and Eqs. (9) and (11), respectively, give

$$\dot{\delta} = \kappa, \tag{17}$$

$$\dot{\kappa} + 2H\kappa = 4\pi G\mu\delta - \frac{k^2}{a^2}\frac{\delta p}{\mu}, \tag{18}$$

where $\delta \equiv \delta\mu/\mu$. Combining these equations we have

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\mu\delta + \frac{k^2}{a^2}\frac{\delta p}{\mu} = 0. \tag{19}$$

This equation is well known in the Newtonian context, see Eq. (15.9.23) in [7], and also valid in Einstein gravity context in the limit of negligible pressure [8]. In the case of CDM we effectively have $\delta p = 0$. However, the equation of state relating δp with $\delta\mu$ in the case of axion is not determined at this point. The equation of state of axion will follow from the perturbed equation of motion.

Under the comoving gauge, Eq. (15) gives

$$\frac{\delta\phi_-}{\phi_{-0}} = \frac{\delta\phi_+}{\phi_{+0}}. \tag{20}$$

Relation between δp and $\delta\mu$ can be determined through α using Eq. (13). We solve Eq. (13) strictly to leading order in H/m as the solution for the background is valid to such an order. Using Eqs. (16) and (17), Eq. (13) gives

$$\alpha = -\frac{1}{2}a^{3/2}\frac{\delta\phi_+}{\phi_{+0}}\frac{k^2}{m^2 a^2}. \tag{21}$$

Eq. (15) gives

$$\delta = 2a^{3/2}\frac{\delta\phi_+}{\phi_{+0}}\left(1 + \frac{1}{4}\frac{k^2}{m^2 a^2}\right). \tag{22}$$

Therefore, we have an equation of state

$$\delta p = \frac{1}{4}\frac{k^2}{m^2 a^2}\frac{1}{1 + \frac{1}{4}\frac{k^2}{m^2 a^2}}\delta\mu. \tag{23}$$

The effective sound speed becomes

$$c_s \equiv \sqrt{\frac{\delta p}{\delta\mu}} = \frac{1}{2}\frac{k}{ma}\left(1 + \frac{1}{4}\frac{k^2}{m^2 a^2}\right)^{-1/2}, \tag{24}$$

which shows an interesting scale dependence; for $k/(ma) \ll 1$ the time dependence $c_s \propto 1/a$ is the same as in the ordinary matter dominated medium, see later. Eq. (19) becomes

$$\ddot{\delta} + 2H\dot{\delta} - \left(4\pi G\mu - \frac{1}{4}\frac{k^4}{m^2 a^4}\frac{1}{1 + \frac{1}{4}\frac{k^2}{m^2 a^2}}\right)\delta = 0. \tag{25}$$

We note that we only have assumed $(H/m)^2 \ll 1$, but have not assumed any condition on $(k/aH)^2$. Thus our equations are valid in all scales. Even in the small scales where the pressure gradient term has a role, in general we have $k^4/(m^2 a^4 H^2) \gg k^2/(m^2 a^2)$, thus ignoring $k^2/(m^2 a^2)$ term we have

$$\ddot{\delta} + 2H\dot{\delta} - \left(4\pi G\mu - \frac{1}{4}\frac{k^4}{m^2 a^4}\right)\delta = 0. \tag{26}$$

This form was presented in Eq. (20) of [4] based on the zero-shear gauge (often termed the conformal Newtonian gauge or the longitudinal gauge). Although Eq. (26), more precisely Eq. (25), in our comoving gauge is valid in all scales, in the zero-shear gauge it is valid only for $k/(aH) \gg 1$; the general density perturbation equation of pressureless medium in the zero-shear gauge is quite complicated, see Eq. (27) in [9]. The competition between gravity and pressure gradient terms in Eq. (26) gives the Jeans scale; assuming a flat axion dominated model we have currently

$$\lambda_J \equiv \frac{2\pi a}{k_J} \equiv 2\pi \left(16\pi G \mu m^2\right)^{-1/4}$$
$$= 5.4 \times 10^{14} cmh^{-1/2} \left(\frac{m}{10^{-5}\,\text{eV}}\right)^{-1/2}, \tag{27}$$

which is quite small corresponding to the Solar System size for $m \sim 10^{-5}$ eV, see also [4]. On scales larger than λ_J, thus effectively in all cosmologically relevant scales, the axion fluid can be regarded as the pressureless ideal fluid. Therefore, the axion is justified as a CDM candidate.

Eq. (26) can be analytically solved for $\Lambda = 0$. For the background, we have $a \propto t^{2/3}$, $H = 2/(3t)$, and $\mu = 1/(6\pi G t^2)$. Eq. (26) has an exact solution

$$\delta_\pm \propto t^{-1/6} J_{\mp 5/2}\left(3At^{-1/3}\right), \quad A \equiv \frac{k^2}{2m}\left(\frac{t^{2/3}}{a}\right)^2. \tag{28}$$

This can be compared with an exact solution in matter dominated era with an equation of state $p \propto \mu^\gamma$ [7]. For $\gamma = 5/3$ we have the solution in Eq. (28) with $A = c_s k t^{4/3}/a$, see Eq. (15.9.41) in [7]; notice that $c_s \propto 1/a$ even in the matter dominated medium. Thus, again we can identify $c_s = k/(2ma)$ as an effective sound speed of the axion fluid.

For $k^2/(maH) \ll 1$ and $\Lambda = 0$ we have a perturbative solution

$$\delta(k, t) = c_+(k) t^{2/3}\left[1 + \frac{3}{8}\frac{k^4}{m^2}\left(\frac{t^{2/3}}{a}\right)^4 t^{-2/3}\right]$$
$$+ c_-(k) t^{-1}\left[1 - \frac{9}{56}\frac{k^4}{m^2}\left(\frac{t^{2/3}}{a}\right)^4 t^{-2/3}\right]. \tag{29}$$

This coincides with the solution in Eq. (25) of [3]. Our previous proof of the axion as a CDM candidate presented in [3] was made based on this and other solutions.

For a clear presentation, in this work we have considered a flat background composed of a single axion component but including the cosmological constant. Our analysis can be easily extended to non-flat case as well as in the presence of additional fluids and fields. We can show that axion behaves as a CDM even in such more realistic situations. For example, in the presence of other components the equation of state in Eq. (23) is valid for the axion component.

Eq. (27) shows an axion Jeans scale where linear perturbation of axion fluid becomes stabilized (oscillates) under that scale. The axion fluid shows peculiar equation of state and effective sound speed presented in Eqs. (23) and (24). Although our neighborhood in the solar system is in a significantly nonlinear stage it is curious to see the possible observational signature of axion fluid based on its contribution to effective pressure. This is left for future investigation.

Recently we have shown that even to the second-order perturbation, the density and velocity perturbation equations of the zero-pressure medium in Einstein's gravity exactly coincide with the ones in Newton's gravity: we call this a relativistic/Newtonian correspondence [10]. The relativistic/Newtonian correspondence to the linear order can be found in Eq. (19) with vanishing pressure. In the present work we have shown that the axion properly treated in relativistic perturbation theory behaves as a pressureless fluid in cosmologically relevant scales, thus justified as a CDM candidate: we may call it the axion/CDM correspondence to the linear order. Whether such a correspondence continues even in the case of second-order perturbation is an interesting open issue at the moment. We will address this important issue in a future occasion.

Acknowledgements

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