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The Perspective Silhouette of a Canal Surface †

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Abstract

We present an efficient and robust algorithm for parameterizing the perspective silhouette of a canal surface and detecting each connected component of the silhouette. A canal surface is the envelope of a moving sphere with varying radius, defined by the trajectory $C(t)$ of its center and a radius function $r(t)$. This moving sphere, $S(t)$, touches the canal surface at a characteristic circle $K(t)$. We decompose the canal surface into a set of characteristic circles, compute the silhouette points on each characteristic circle, and then parameterize the silhouette curve. The perspective silhouette of the sphere $S(t)$ from a given viewpoint consists of a circle $Q(t)$; by identifying the values of t at which $K(t)$ and $Q(t)$ touch, we can find all the connected components of the silhouette curve of the canal surface.

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Three Dimensional Graphics and Realism

1. Introduction

Silhouette curves provide a visual cue to the shape of an object and have many practical applications: for instance, in computing the visible area of an object, removing hidden curves, back-face culling and non-photorealistic rendering [1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 16, 17]. There are two kinds of silhouette curve: parallel silhouettes correspond to idealized viewpoints at an infinite distance from the object; perspective silhouettes correspond to real viewpoints at finite locations. Generally, the shapes of a parallel silhouette and a perspective silhouette are quite different, and it is more difficult to compute the perspective silhouette than the parallel silhouette.

Markosian et al. [12] used a randomized algorithm to find silhouettes of polyhedral models. They found a silhouette edge by examining only a small subset of the edges in the model, and then traced the entire silhouette from an initial edge. Benichou and Elber [1] computed parallel silhouettes of polygonal objects using the Gaussian sphere. They mapped the normal vectors of edges of the object and the view direction on to the Gaussian sphere, where they become great arcs and a great circle. Then, they projected that data from the Gaussian sphere on to a circumscribing cube, where the arcs and the circle on the Gaussian sphere become straight lines. They computed the silhouette curve by intersecting these lines. Krishnan and Manocha [11] computed the parallel silhouette of a free-form surface by using the marching method to trace the silhouette curves from patch boundaries.

Kim and Lee [10] presented a method for computing the parallel silhouette of surfaces of revolution and canal surfaces. They utilized the fact that both these types of surface can be decomposed into a set of circles, and the normal vectors of

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these circles form a cone. Using these characteristics, they computed the parallel silhouettes of the surfaces.

The formulation of the perspective silhouette of a parametric surface $S(u,v)$ from the viewpoint $\vec{O}$ is well-known. The silhouette consists of a set of surface points which satisfy

$$(S(u,v)-\vec{O})\cdot\vec{N}(u,v)=0,$$

where $\vec{N}(u,v)$ is a surface normal. Although the equation is not difficult to define, tracing the curve represented by this equation is difficult because the equation is an implicit form, usually of high degree.

In this paper, we present an efficient and robust algorithm for parameterizing the perspective silhouette of a canal surface, and detecting all the connected components of the silhouette. A canal surface is the envelope of a moving sphere with varying radius, and is useful for representing long thin objects: for instance, pipes, poles, ropes, 3D fonts, brass instruments, or internal organs of the body [14]. Canal surfaces are also frequently used in solid and surface modeling for CAD/CAM. Representative examples are natural quadrics, tori, pipe surfaces, and Dupin cyclides [13, 15].

For computer graphics and animation, models consisting of canal surfaces have several advantages:

1. Rendering of the model can be done efficiently [14, 18].
2. Computing the distance between models (and hence detecting collisions) is relatively rapid.
3. Construction of the model is easy.
4. The geometric information to represent the model only requires a small amount of space.

However, there are definite limits to the shapes that can be modeled with canal surfaces.

A canal surface is defined by the center trajectory $C(t)$ and radius function $r(t)$ of a moving sphere. This moving sphere, $S(t)$, touches the canal surface at a characteristic circle $K(t)$. We decompose the canal surface into a set of characteristic circles, compute the silhouette points on each characteristic circle, and then parameterize the silhouette curve. The perspective silhouette of the sphere $S(t)$ from a given viewpoint consists of a circle $Q(t)$. By identifying the values of t at which $K(t)$ and $Q(t)$ touch, we can find all the connected components of the silhouette curve of the canal surface.

This paper is organized as follows. In Section 2, we show how to decompose a canal surface into a set of circles, and then represent the surface in a parametric form. We parameterize the perspective silhouette of a canal surface in Section 3. In Section 4, we propose two methods to find each connected component of the perspective silhouette. Then, in Section 5, we present an algorithm to compute the perspective silhouette and Section 6 contains some examples. We conclude this paper in Section 7.

2. Parameterization of a Canal Surface

In this section, we parameterize a given canal surface by decomposing it into a set of characteristic circles. Let us denote the spine curve and the radius function of a canal surface as $C(t)$ and $r(t)$. Then, a surface point $\vec{p}=(x,y,z)$, which is on the moving sphere centered at $C(t)$ with radius $r(t)$, satisfies the following property:

$$\|\vec{p}-C(t)\|^2-r(t)^2=0. \quad (1)$$

The point $\vec{p}$ is on the envelope surface of the moving sphere, so the following property is also satisfied:

$$(\vec{p}-C(t))\cdot C'(t)+r(t)r'(t)=0. \quad (2)$$

Equations (1) and (2) define a canal surface.

If $\alpha(t)$ is the angle between two vectors $\vec{p}-C(t)$ and $C'(t)$, the following equation can be derived from Equations (1) and (2) (see Figure 1):

$$\cos\alpha(t) = \frac{(\vec{p}-C(t))\cdot C'(t)}{\|\vec{p}-C(t)\|\|C'(t)\|} = -\frac{r'(t)}{\|C'(t)\|}. \quad (3)$$

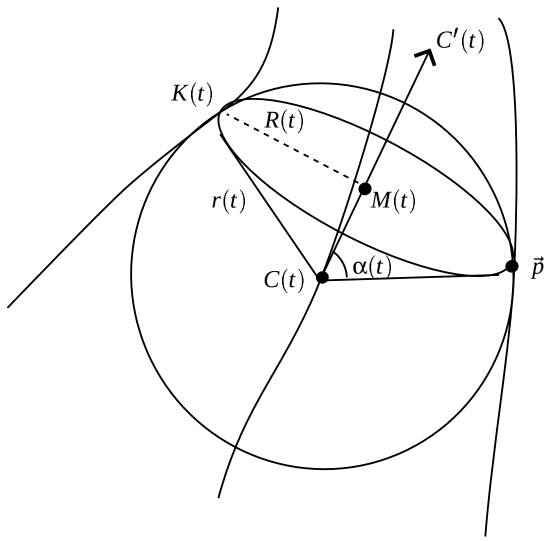

Figure 1: *A circle $K(t)$ on the sphere $S(t)$.*

Let us denote the moving sphere with center $C(t)$ and radius $r(t)$ as $S(t)$. If $S(t)$ touches the canal surface at a circle $K(t)$, then the normal vector of the plane which contains $K(t)$ is $C'(t)$. Using Equation (3), we compute the center $M(t)$ and radius $R(t)$ of the circle $K(t)$ as follows:

$$M(t) = C(t)+r(t)\cos\alpha(t)\frac{C'(t)}{\|C'(t)\|}$$

$$= C(t) - r(t)r'(t)\frac{C'(t)}{||C'(t)||^2}$$

$$R(t) = r(t)\sin\alpha(t) = r(t)\frac{\sqrt{||C'(t)||^2 - r'(t)^2}}{||C'(t)||}.$$

Then, the canal surface is parameterized as

$$K(t,\theta) = M(t) + R(t)(\cos\theta\vec{b}_1(t) + \sin\theta\vec{b}_2(t)),$$

where $0 \le \theta < 2\pi$, and $\vec{b}_1(t)$, $\vec{b}_2(t)$ are the basis vectors of the plane which contains $K(t)$:

$$\vec{b}_1(t) = \frac{C'(t) \times C''(t)}{||C'(t) \times C''(t)||}$$

$$\vec{b}_2(t) = \frac{C'(t) \times \vec{b}_1(t)}{||C'(t) \times \vec{b}_1(t)||}.$$

3. Parameterization of the Perspective Silhouette

Given a canal surface with spine curve $C(t)$ and radius function $r(t)$, $t_{min} \le t \le t_{max}$, let this be a regular surface. That is, we assume that $r(t) > 0$, $||C'(t)||^2 > r'(t)^2$, and $C(t)$ is C^2-continuous. If $K(t,\theta)$ is the parametric representation of given canal surface, we denote the normal vector of $K(t,\theta)$ as $\vec{N}(t,\theta)$. From a given viewpoint $\vec{O} = (O_x, O_y, O_z)$, the perspective silhouette of the canal surface is the set of points which satisfy

$$\vec{N}(t,\theta) \cdot (K(t,\theta) - \vec{O}) = 0.$$

Usually, the normal vector of a surface $K(t,\theta)$ may be computed from the following equation:

$$\vec{N}(t,\theta) = \frac{\partial K(t,\theta)}{\partial t} \times \frac{\partial K(t,\theta)}{\partial \theta}.$$

However, for a canal surface, we may simplify the representation of the normal vector so that it becomes

$$\vec{N}(t,\theta) = K(t,\theta) - C(t).$$

The actual degree of this equation is usually lower than that implied by the vector product.

The equation of the perspective silhouette of $K(t,\theta)$ may be expanded as follows:

$$(K(t,\theta) - C(t)) \cdot (K(t,\theta) - \vec{O}) = 0.$$

By replacing $K(t,\theta)$ with $M(t) + R(t)(\cos\theta\vec{b}_1(t) + \sin\theta\vec{b}_2(t))$, the following equation is derived:

$$A(t)\cos\theta + B(t)\sin\theta + D(t) = 0, \tag{4}$$

where

$$A(t) = \vec{b}_1(t) \cdot (C(t) - \vec{O})$$

$$B(t) = \vec{b}_2(t) \cdot (C(t) - \vec{O})$$

$$D(t) = \frac{-r'(t)(C(t) - \vec{O}) \cdot C'(t) + r(t)||C'(t)||^2}{||C'(t)||\sqrt{||C'(t)||^2 - r'(t)^2}}.$$

Using Equation (4), we can derive the following formula for the functions of θ in Equation (4) :

$$\cos\theta = \frac{-A(t)D(t) \pm B(t)\sqrt{A(t)^2 + B(t)^2 - D(t)^2}}{A(t)^2 + B(t)^2}$$

$$\sin\theta = \frac{-A(t)\cos\theta - D(t)}{B(t)}.$$

Substituting back, we are able to parameterize the perspective silhouette of the canal surface as follows:

$$\vec{p}(t) = M(t) + R(t)(c(t)\vec{b}_1(t) + s(t)\vec{b}_2(t)), \tag{5}$$

where

$$c(t) = \frac{-A(t)D(t) \pm B(t)\sqrt{A(t)^2 + B(t)^2 - D(t)^2}}{A(t)^2 + B(t)^2}$$

$$s(t) = \frac{-A(t)c(t) - D(t)}{B(t)}.$$

4. Detecting each Connected Component

In Section 3, we derived the parametric representation of the perspective silhouette curve of a given canal surface, $\vec{p}(t)$ (Equation (5)). If we compute a set of points $\vec{p}(t)$ by varying the value of the parameter t, and connect them, then the components of the silhouette are traced. The value of $A(t_*)^2 + B(t_*)^2 - D(t_*)^2$ determines whether the circle $K(t_*,\theta)$ contains a silhouette point or not. If $A(t_*)^2 + B(t_*)^2 - D(t_*)^2$ is positive, then $K(t_*,\theta)$ contains two silhouette points; if it is negative, $K(t_*,\theta)$ contains none.

Functions $A(t)$, $B(t)$, and $D(t)$ are continuous; thus, $A(t)^2 + B(t)^2 - D(t)^2$ is also a continuous function. If there are two values t_0 and t_1, such that $t_{min} \le t_0, t_1 \le t_{max}$, and which also satisfy $A(t_0)^2 + B(t_0)^2 - D(t_0)^2 < 0$ and $A(t_1)^2 + B(t_1)^2 - D(t_1)^2 > 0$, then there exists a value t_m between t_0 and t_1 such that $A(t_m)^2 + B(t_m)^2 - D(t_m)^2 = 0$. Therefore the solutions of t which satisfy $A(t)^2 + B(t)^2 - D(t)^2 = 0$ represent the boundary values of t for the connected components of the silhouette.

In order to find each connected component of the perspective silhouette, we need a way to solve $A(t)^2 + B(t)^2 - D(t)^2 = 0$. This equation is expanded as follows:

$$\begin{aligned} &||C'(t)||^2(||C'(t)||^2 - r'(t)^2) \\ &((\vec{b}_1(t) \cdot (C(t) - \vec{O}))^2 + (\vec{b}_2(t) \cdot (C(t) - \vec{O}))^2) \\ &-(-r'(t)(C(t) - \vec{O}) \cdot C'(t) + r(t)||C'(t)||^2)^2 = 0. \end{aligned} \tag{6}$$

If the basis vectors $\vec{b}_1(t)$ and $\vec{b}_2(t)$ of the characteristic circles are represented as equations of low degree, then Equation (6) may be solved without great difficulty. However, the vectors $\vec{b}_1(t)$ and $\vec{b}_2(t)$ contain cross-product and square root terms, so the degree of this equation is usually high. We now introduce a method specifically for such cases.

On a canal surface $K(t,\theta)$, the characteristic circle $K(t_*,\theta)$ is embedded in a sphere $S(t_*)$ with center $C(t_*)$ and

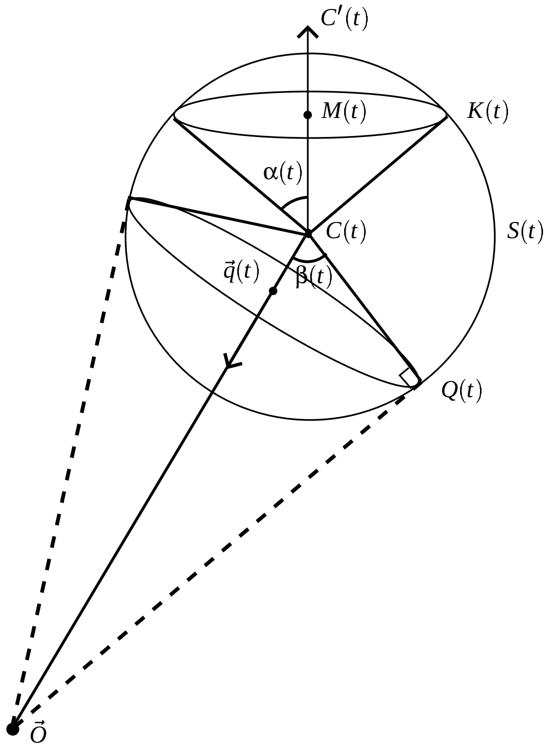

Figure 2: *Circles $K(t)$ and $Q(t)$ on a sphere $S(t)$.*

radius $r(t_*)$. From a given viewpoint $\vec{O}$, the perspective silhouette of $S(t_*)$ may be computed as follows. Let $\vec{p}$ denote an arbitrary point on $S(t_*)$, and let $\vec{N}_p$ denote the normal vector of $S(t_*)$ at $\vec{p}$. Then, $\vec{N}_p$ can be represented as

$$\vec{N}_p = \vec{p} - C(t_*)$$

and the perspective silhouette of $S(t_*)$ consists of the points which satisfy

$$(\vec{p} - \vec{O}) \cdot \vec{N}_p = (\vec{p} - \vec{O}) \cdot (\vec{p} - C(t_*)) = 0.$$

Notice that $(\vec{x} - \vec{O}) \cdot (\vec{x} - C(t_*)) = 0$, where $\vec{x} \in \boldsymbol{R}^3$, represents a sphere with center $(\vec{O} + C(t_*))/2$ and radius $\|\vec{O} - C(t_*)\|/2$. Thus, the point $\vec{p}$ is embedded in the intersection of two spheres, and the locus of $\vec{p}$ forms a circle. We denote this circle as $Q(t_*)$ (see Figure 2).

Let us consider the silhouette points on the canal surface as the intersection of two circles. The silhouette points lie on the sphere $S(t)$. All points on the canal surface that lie on $S(t)$ are embedded in the circle $K(t)$, so the silhouette points are on $K(t)$. From the viewpoint $\vec{O}$, the silhouette point on $S(t)$ is embedded in $Q(t)$. Thus, among the points on $S(t)$, the silhouette points of the canal surface consist of the intersection between $K(t)$ and $Q(t)$ (see Figure 3(a)).

In order to compute the silhouette points by varying the value of t continuously, let us assume that $K(t) \cap Q(t) \neq \emptyset$, where $t_{i-1} \leq t \leq t_i$; and $K(t) \cap Q(t) = \emptyset$, where $t_{i-2} < t < t_{i-1}$ and $t_i < t < t_{i+1}$. Then, for $t_{i-1} \leq t \leq t_i$, the intersection between $K(t)$ and $Q(t)$ comprises one connected component of the silhouette (Figure 3(b)). In this case, for $t \in \{t_{i-1}, t_i\}$, the circles $K(t)$ and $Q(t)$ touch at a single point. The value of t at each such point is a boundary value of one of the connected components of the canal surface silhouette.

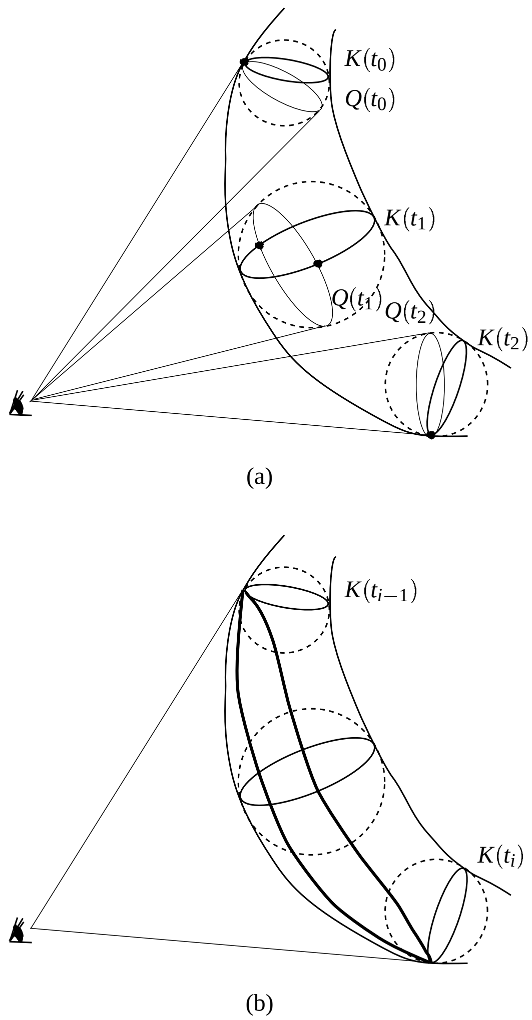

Figure 3: *A silhouette curve consists of $K(t) \cap Q(t)$.*

The relative position of two circles $K(t_*)$ and $Q(t_*)$ may be classified into three cases: i) they intersect at two points, ii) they do not intersect, and iii) they intersect tangentially. When $K(t_*)$ and $Q(t_*)$ intersect at two points, these points are embedded in the silhouette of the canal surface (Figure 4(a)). If $K(t_*)$ and $Q(t_*)$ do not intersect, then $K(t_*)$ does not contain any silhouette points (Figure 4(b)). In the last case, $K(t_*)$ and $Q(t_*)$ touch at a point, or the two circles are coincident (Figure 4(c)).

We may detect the cases when $K(t)$ and $Q(t)$ touch by considering the cones which contain $K(t)$ and $Q(t)$, respectively. The surface normals at points on $K(t_*, \theta)$ form a cone $\Gamma(t_*)$ with its vertex at $C(t_*)$ and its axis parallel to $C'(t_*)$. The half-angle of $\Gamma(t_*)$, $\alpha(t_*)$, satisfies Equation (3) (please refer to Section 2 and Figure 1). Circle $K(t_*, \theta)$ is embedded in the sphere $S(t_*)$ with center $C(t_*)$ and radius $r(t_*)$. Let us denote the cone with vertex $C(t_*)$, and containing $Q(t_*)$, as $\Gamma_q(t_*)$. The axis of $\Gamma_q(t_*)$ is parallel to $\vec{O} - C(t_*)$. If we denote the half-angle of the cone as $\beta(t_*)$, then the following relation is satisfied (see Figure 2):

$$\cos\beta(t) = \frac{r(t)}{\|\vec{O} - C(t)\|}. \qquad (7)$$

The necessary and sufficient condition for $K(t)$ and $Q(t)$ to have a tangent intersection is that the two cones $\Gamma(t)$ and $\Gamma_q(t)$ touch, and this condition is represented by the equation

$$\gamma(t) = |\alpha(t) \pm \beta(t)|,$$

where $\gamma(t)$ is the angle between the axes of $\Gamma(t)$ and $\Gamma_q(t)$ (i.e. between $C'(t)$ and $\vec{O} - C(t)$), and $0 \leq \gamma(t) < \pi$ (see Figure 4(c)). We can now derive the following equation:

$$\cos\gamma(t) = \cos(\alpha(t) \pm \beta(t)) = \cos\alpha(t)\cos\beta(t) \pm \sin\alpha(t)\sin\beta(t).$$

Using Equations (3), (7), and $\cos\gamma(t) = \dfrac{C'(t)\cdot(\vec{O} - C(t))}{\|C'(t)\|\|\vec{O} - C(t)\|}$, this equation can be expanded to become :

$$\begin{aligned}&((\vec{O} - C(t))\cdot C'(t))^2 + 2r(t)r'(t)(\vec{O} - C(t))\cdot C'(t)\\ &-\|\vec{O} - C(t)\|^2(\|C'(t)\|^2 - r'(t)^2)\\ &+\|C'(t)\|^2 r(t)^2 = 0. \qquad (8)\end{aligned}$$

The solution of Equation (8) provides the boundary values of t for each connected component. Compared to the degree of Equation (6), that of Equation (8) is lower, so Equation (8) provides the better solution to find the boundary values of t.

5. Algorithm

In the algorithm **Persp_Silhouette_of_Canal_Surface**, we have implemented a method of computing perspective silhouette of a given canal surface presented in Sections 3 and 4.

6. Examples

In this section, we present some examples of computing the perspective silhouettes of canal surfaces.

Helical Surfaces

We may represent a helical surface as a canal surface with the spine curve $C(t) = (\Lambda\cos t, \Lambda\sin t, kt)$ and the radius

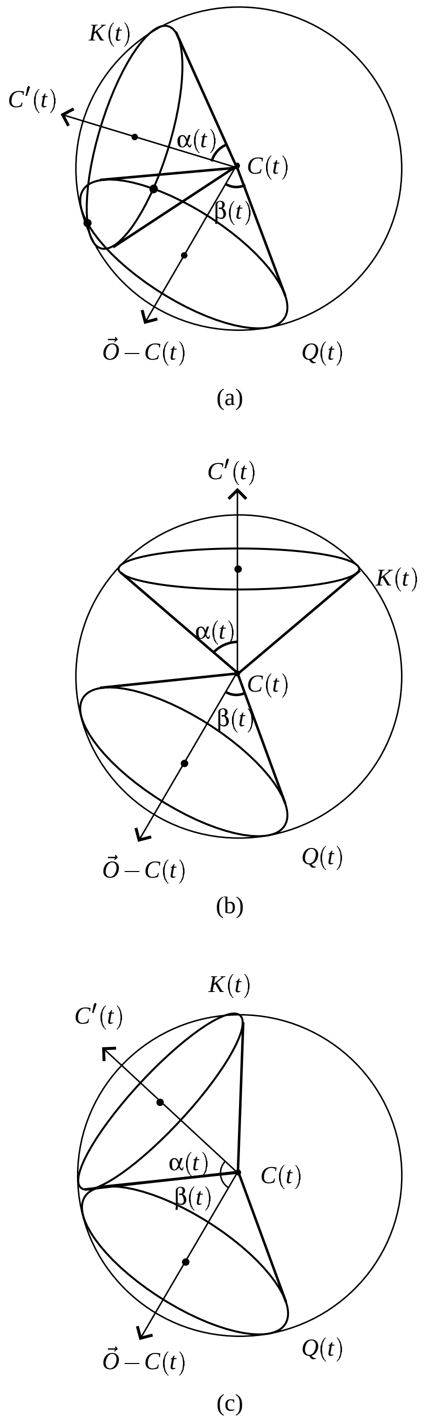

Figure 4: *The relative position of two circles $Q(t)$ and $K(t)$.*

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Algorithm:
Persp_Silhouette_of_Canal_Surface

Input:
   C(t) = (x(t), y(t), z(t)),        /* spine curve */
   r(t),                          /* radius function */
   O⃗ = (O_x, O_y, O_z),                /* viewpoint */

begin
  /* degenerate case */
  for each t_* ∈ { t | A(t) = 0 and B(t) = 0 } do
   if D(t_*) = 0 then
    draw a circle K(t_*, θ), 0 ≤ θ < 2π;

  /* generic case */
  T = {t_min, t_max} ∪ { t |
   ((O⃗ − C(t)) · C'(t))^2 + 2r(t)r'(t)(O⃗ − C(t)) · C'(t)
   −||O⃗ − C(t)||^2(||C'(t)||^2 − r'(t)^2) + ||C'(t)||^2 r(t)^2 = 0};
  sort t values in T : T = { t_i | 0 ≤ i < n };
  for i = 1 to n−1 do begin
   t_* = (t_{i−1} + t_i)/2;
   if A(t_*)^2 + B(t_*)^2 − D(t_*)^2 ≥ 0 then begin
    for t_{i−1} ≤ t ≤ t_i, draw a curve
    M(t) + R(t)(c_a b⃗_1(t) + s_a b⃗_2(t)), where
     c_a = (−A(t)D(t) + B(t) sqrt(A(t)^2 + B(t)^2 − D(t)^2)) / (A(t)^2 + B(t)^2),
     and s_a = (−A(t)c_a − D(t))/B(t);
    for t_{i−1} ≤ t ≤ t_i, draw a curve
    M(t) + R(t)(c_b b⃗_1(t) + s_b b⃗_2(t)), where
     c_b = (−A(t)D(t) − B(t) sqrt(A(t)^2 + B(t)^2 − D(t)^2)) / (A(t)^2 + B(t)^2),
     and s_b = (−A(t)c_b − D(t))/B(t);
   end
  end
end
```

function $r(t) = \lambda$. The center and radius of each characteristic circle may be written as

$$M(t) = (\Lambda \cos t, \Lambda \sin t, kt)$$
$$R(t) = \lambda$$

with the basis vectors

$$\vec{b}_1(t) = \frac{(k \sin t, -k \cos t, \Lambda)}{\sqrt{\Lambda^2 + k^2}}$$
$$\vec{b}_2(t) = (\cos t, \sin t, 0).$$

We assume that the viewpoint is $\vec{O} = (O_x, O_y, O_z)$. Then the perspective silhouette of the helical surface is represented by the implicit equation

$$A(t) \cos\theta + B(t) \sin\theta + D(t) = 0,$$

where

$$A(t) = \frac{1}{\sqrt{\Lambda^2 + k^2}}(\Lambda kt - O_x k \sin t + O_y k \cos t - O_z \Lambda)$$
$$B(t) = \Lambda - (O_x \cos t + O_y \sin t)$$
$$D(t) = \lambda.$$

The implicit equation $A(t)^2 + B(t)^2 - D(t)^2 = 0$, which represents the boundary values of t for each connected component of the silhouette, can now be derived :

$$(\Lambda - O_x \cos t - O_y \sin t)^2 + \frac{1}{\Lambda^2 + k^2}(\Lambda kt - O_x k \sin t + O_y k \cos t - O_z \Lambda)^2 - \lambda^2 = 0.$$

Figure 5 shows an example of a perspective silhouette computed for a helical surface.

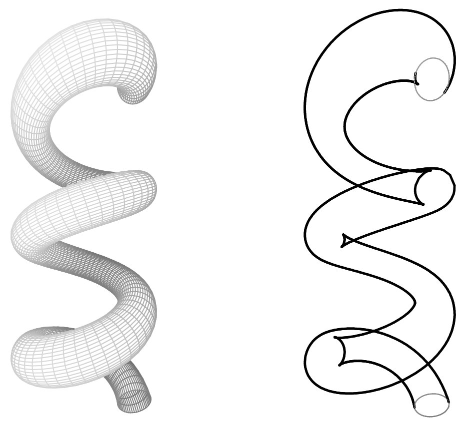

Figure 5: *The perspective silhouette of a helical surface.*

The Dupin Cyclide

A Dupin cylide

$$(x^2 + y^2 + z^2 - \mu^2 + b^2)^2 - 4(ax - c\mu)^2 - 4b^2y^2 = 0,$$

where $a^2 = b^2 + c^2$, $a, b > 0$ and $c, \mu \geq 0$, may be represented as a canal surface in the following form [9]:

$$C(t) = (a \cos t, b \sin t, 0)$$
$$r(t) = -c \cos t + \mu.$$

The center trajectory and the radius of the characteristic circles are

$$M(t) = (a \cos t, b \sin t, 0) - c \sin t(-c \cos t + \mu)\frac{(-a \sin t, b \cos t, 0)}{a^2 \sin^2 t + b^2 \cos^2 t}$$
$$R(t) = \frac{b(-c \cos t + \mu)}{\sqrt{a^2 \sin^2 t + b^2 \cos^2 t}},$$

with the basis vectors

$$\vec{b}_1(t) = (0,0,1)$$
$$\vec{b}_2(t) = \frac{(b\cos t, a\sin t, 0)}{\sqrt{a^2\sin^2 t + b^2\cos^2 t}}.$$

With using the condition $a^2 = b^2 + c^2$, the functions $A(t)$, $B(t)$, and $D(t)$ may then be derived :

$$\begin{aligned} A(t) &= -O_z \\ B(t) &= (ab - O_x b\cos t - O_y a\sin t)/\sqrt{a^2 - c^2\cos^2 t} \\ D(t) &= (c\cos t(-b^2 - \mu c\cos t + O_y b\sin t + O_x a\cos t) \\ &\quad + a(\mu a - O_x c))/(b\sqrt{a^2 - c^2\cos^2 t}). \end{aligned}$$

To compute the values of t which correspond to the boundary of each connected component, we may use either the equation $A(t)^2 + B(t)^2 - D(t)^2 = 0$ or Equation (8). When we compare the degree of these two equations, we find that the degree of $A(t)^2 + B(t)^2 - D(t)^2 = 0$ is 12, but that of Equation (8) is only 8. It is therefore easier to solve Equation (8), and this equation can then be expanded :

$$\begin{aligned} &(-O_x a\sin t + O_y b\cos t + c^2\cos t\sin t) \\ &(-O_x a\sin t + O_y b\cos t - c^2\cos t\sin t + 2\mu c\sin t) \\ &-b^2(\|\vec{O}\|^2 - 2O_x a\cos t - 2O_y b\sin t + b^2 + 2\mu c\cos t - \mu^2) \\ &+c^2\sin^2 t(c^2\cos^2 t - 2\mu c\cos t + \mu^2) = 0. \end{aligned}$$

Figure 6 shows an example of a perspective silhouette computed for a Dupin cyclide.

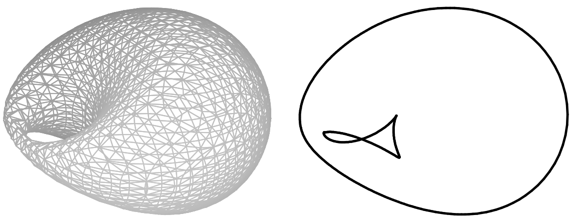

Figure 6: *The perspective silhouette of a Dupin cyclide.*

Other Examples

Figure 7 shows examples of the perspective silhouette of three-dimensional models consisting of canal surfaces only. The degree of the spine curves and radius functions of the canal surfaces in these models are a maximum of three.

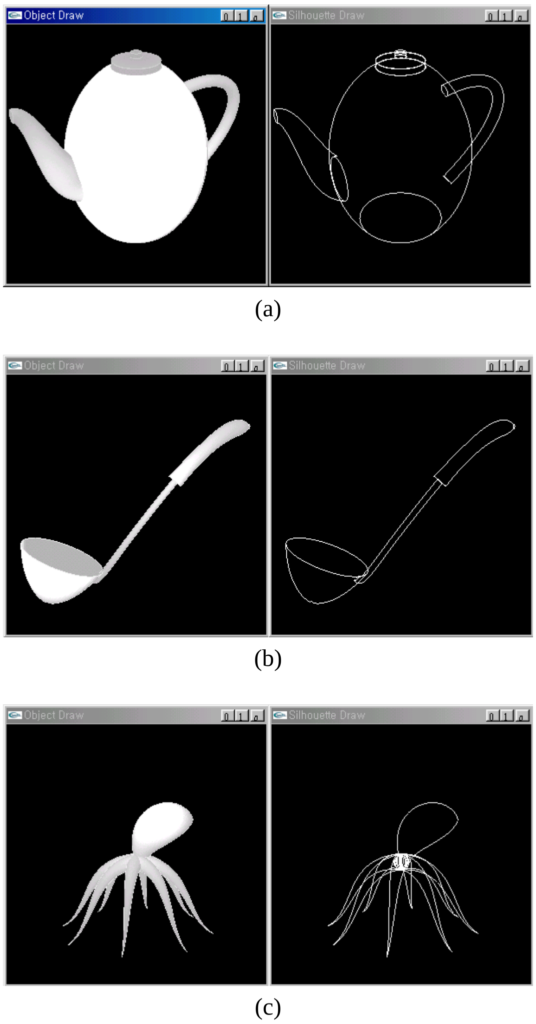

Figure 7: *The perspective silhouettes of a teapot, a scoop, and an octopus.*

7. Conclusions

We have presented a method to compute the perspective silhouette of a canal surface. We proposed a method to parameterize the perspective silhouette of a canal surface and presented an efficient and robust method to find all the connected components of a silhouette.

A canal surface is the envelope of a moving sphere with varying radius. We decomposed a canal surface into a set of circles, and computed the silhouette points on each circle. To find each connected component of the silhouette curve, we utilized two circles defined on the moving sphere $S(t)$, which contains the perspective silhouette as a circle and also a characteristic circle of the canal surface. The intersection of these two circles is embedded in the perspective silhouette of the canal surface. By computing the values of t at which the two circles touch, we were able to find all the connected components of the perspective silhouette of the canal surface.

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